Optimization of the cost of urban traffic through an online bidding platform for commuters

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ABSTRACT

In this paper, we consider the problem of increasing efficiency of a transportation system through optimizing the behaviour of commuters. The assumption is that the time spent in the traffic can be represented by a monetary value and hence introduction of monetary compensations can lead to a more efficient organization of the transportation system. In our model, heterogeneous travelers differently assess the value of their time spent in congestion, hence it is presumably viable to reduce traffic in the most congested streets by introducing a bidding mechanism that will allow the participants who have a lower monetary value of time to receive a compensation financed by the group of commuters that have a higher value of time spend in congestion. We start by presenting a design of a bidding system for optimal allocation of traffic. We analyze the properties of the proposed algorithm and show that it leads to a more efficient allocation of vehicles than the theoretical allocation that could be achieved in the Nash Equilibrium of an uncontrolled transportation network. Subsequently, we verify the proposed auction design via an agent-based simulation model representing the Manhattan area of New York City. The results of our simulation confirm theoretical findings that the introduction of the proposed auction mechanism in a real city settings leads to a more efficient allocation of routes or means of transportation chosen by commuters.

1. Introduction

Over the last several decades, a strengthening trend can be observed in modern societies where an increasing number of people migrate from the country towards large cities in search of better paid occupation and higher quality of life. This tendency leads to a number of new urban management issues which local authorities must take into consideration. One of these issues is the occurrence of traffic congestion during rush hour times.

It is a natural tendency in cities that most of the commercial and business areas are located in the city center, while the majority of residential and living zones are placed on the outskirts. Naturally, this results in the emergence of an intense one-direction flow of commuters driving towards the city center early in the morning and coming back from work later in the evening.

Due to a limited road capacity and an increasing demand for fast relocation capability, the competition for lane occupation has been increasing. Access to public streets is usually without barriers and scarcely regulated. Thus, space rationing is for the most part based on a simple rule: *first come first served*. In these conditions, commuters have no incentive to choose any route other than the shortest path to their destination point. This, in turn, results in an excessive demand for road space over the available capacity and hence an emergence of congestion in bottle-neck lanes. Extended time spent commuting to work, equally for all participants of road traffic and regardless of their value of time, will most likely lead to an ineffective distribution of resources such as road capacity and spare time.

The issues of sub-optimal road space rationing and street overcrowding during peak hours are often of concern to both local authorities and researchers working on the economics of transportation and traffic modelling. In practice, the approach most commonly used to solve these kinds of problems is simple tolling of selected streets at particular hours of the day. Much research has been conducted and a number of solutions have been proposed regarding how to determine the optimal charges and best sets of roads to be tolled—for further information see e.g., [30] or [18].

Although imposing fixed charges on certain roads may seem like a fairly straightforward solution, it nonetheless lacks broader flexibility, as it involves undertaking high-level political decisions which usually require a lot of time in

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order to get approved. Furthermore, there is always a risk that the tolls will neither be optimal nor improve traffic to the expected extent. In addition, these decisions may negatively affect the public perception of authorities, since imposing additional taxes or charges is usually not very popular among local constituencies. On the other hand, taxes collected via road tolling systems can later be used for the purposes of local community and infrastructural investments such as the construction of new interchanges, the enhancement of road capacities, and the maintenance of existing streets or subsidies for alternative means of transport—for more information on tax influence see, for example, [1].

Internet of Things (IoT) creates new possibilities in terms of introducing more dynamic and modern forms of road pricing and traffic control. The idea of real-time communication between traffic participants and road infrastructure, i.e., Vehicle to Infrastructure (IV2) communication, has been already considered as a way to reduce traffic congestion—e.g., by syncing vehicle transit with semaphores to create fast-travel corridors through a city [23] or by ahead reservation of lane space.

However, instead of focusing on searching for optimal tolls or improving infrastructure performance, in this article we consider the possibility of developing a system which would aim to improve the position of each participant without interfering with the local infrastructure itself. This can be achieved by assuming that commuters have their own preferences about the value of time spent in congestion. Therefore, some of them may be willing to pay to be able to travel faster than they would normally do, while other drivers may agree to choose longer paths in exchange for some monetary benefits.

In our reference scenario (i.e., where no extra incentives are included), since agents have no incentive to cooperate, we may assume that the entire system converges to Nash Equilibrium (NE) [20]. This means that no single agent is willing to change its taken route, since it would entail worsening their utility. However, reaching Nash Equilibrium does not necessarily mean simultaneously achieving Pareto efficiency (for example, as in the classic *prisoner's dilemma* [3]). Thus, we are trying to answer the question of whether it is possible to find better (in terms of Pareto) allocation of money and time for the commuters by using external incentives such as mutual payments, while taking advantage of capabilities of modern vehicles equipped with direct communication protocols and embedded software systems or mobile applications. The proposed algorithm can be applied to optimize commuter flow between alternative routes as well as between private and public transportation systems. We develop this model using a static flow congestion model [31]. Note that the flow-based approach simplifies the complexity of the congestion mechanism (compared to a queuing based approach). However, since we focus on the auction design, this seems to be a reasonable approach.

2. Literature review

Problems of optimal route selection and traffic congestion pricing in urban areas are widely known and described in a number of articles and monographs. A concept of space rationing has been already raised in [21] and [14]. Later on, books *Studies in the Economics of Transportation* [4] and *Traffic Flow Theory* [8] have established foundations for quantitative, statistical, and spatial simulations of urban traffic by describing issues such as traffic stream modeling, traveling time estimation, road capacities, train scheduling, or predicting of commuters' decisions and behavior.

Further studies have introduced and developed multiple approaches to the urban traffic modelling. Commonly used algorithms based on finding the shortest paths within a graph were described in [6]. However, over time more sophisticated approaches (often originating from Dijkstra's algorithm) have been proposed in regard to urban route selection. For example, [22] provides an overview of the state of the art methods in the analysis of route choice behavior focusing on the discrete choice modeling framework. Prato also distinguishes three basic types of models: deterministic, stochastic, and dynamic traffic assignment.

A related topic—although regarding mass transit commuters—has been raised by [28]. In their article, authors propose fare scheme aimed to improve Pareto efficiency of heterogeneous commuters by relieving congestion during peak hours via influencing some passengers departure time choices. However, instead of trying to influence commuters travel time decisions, we propose mechanisms to optimize spatial dimension decisions, i.e., routes taken by each agent during the rush hours.

[23] made a review of 41 published research papers on optimal control theory in Intelligent Transportation Systems (ITS). ITS focuses on real life implementation of intelligent networks of communicating devices such as vehicles, roads, traffic lights, etc., which collectively create a system capable to make more systematic decisions in order to solve current transportation problems, such as congestion reduction. The other issue raised in this paper, that is strongly related to the optimal route finding problem, is a congestion pricing problem. Due to limited capacity of roads and rapidly increasing number of commuters, especially in large city centers, it becomes necessary to introduce tolls for drivers that would

reflect actual externalities caused by urban traffic, such as extra time spent in a congestion. Numerous articles have been devoted to the problem of optimal toll assessment and its implementation. For example, [30] present a market-inspired approach to reservation-based urban road traffic management, which aims to extend the idea from [7] of an advanced urban road traffic infrastructure that allows for cars to individually reserve space and time at an intersection so as to be able to cross it safely and quickly.

[18] review economic principles of congestion pricing, focusing on complications of various models which can lead to their limited relevance. A brief history of the process of introducing Downtown Congestion Pricing can be found in [17]. The authors gathered experiences from applying charging zones and road tolls in several city centers. The paper compares different approaches to the congestion pricing system design across the analyzed cities and their effectiveness on the basis of historical traffic data. A comprehensive review of different congestion pricing methodologies has been presented in [5]. The paper describes and compares various approaches and technologies which have been applied in numerous cities and countries and shows that combining several ITS technologies, such as Automated Number Plate Recognition, V2V (e.g., via Dedicated Short Range Communications), V2I (via cellular networks) can be combined into a single commuter friendly system.

[12] present a simulation-based approximation algorithm to calculate dynamic tolls under the dynamic marginal cost pricing (MCP) scheme, which is a direct extension of the static MCP [15]. Due to the complexity of an actual marginal-cost dynamic pricing policy calculation, an approximate method was used, which assumed link-based marginal cost estimation in order to compute time-dependent tolls updated every 10 minutes. Another practical implementation of MCP theory has been described in [26]. This work presents two alternative time-varying cordon-based tolls applied to a complex urban network characterized by heterogeneous features in space (different typologies of roads) and time (traffic demand patterns within the day). Moreover, an agent-based model was used to analyze the congestion patterns in the large urban networks and the distribution of traffic influence on effectiveness of the cordon-based tolls.

An agent-based approach to geospatial modeling has been presented by [32]. In this paper, heterogeneous agents are characterized by different personal assessment of the Value of Time, who explore the road network in search of better paths to their predefined activities. The agents are able to exchange information and learn new traveling patterns. The authors assume that the road network is a mixture of public and private roads, so the agents are able to choose between tolled and untolled rides depending on their personal preferences of time and money. The goal of the paper was to simultaneously model the toll and capacity choices on general transportation networks and to investigate its influence on overall social economic welfare benefits and distribution effects.

An interesting approach using multiagent modeling of congestion pricing has been presented in [27]. The authors have studied an impact of different pricing strategies on the social welfare, by modeling traffic simulation of tens of thousands of self-driving vehicles travelling through Austin, Texas network. The study compares more traditional pricing schemes, such as distance-based or link-based schemes, as well as more advanced congestion pricing strategies such as dynamic marginal cost or travel time-congestion-based pricing. Another use of multiagent-based simulations have been studied in [9]. The article analyzes how multiagent architectures can be applied to the problem of strategic road traffic management. Two different knowledge-based intelligent traffic management systems have been described and compared on an example of street network of Barcelona. The results have demonstrated that the use of artificial intelligence techniques to develop traffic management systems provides an evident added value to conventional systems.

In our article, however, we focus on a possibility to allow heterogeneous agents to trade time spent on commuting for cash. Instead of arbitrarily tolling particular roads or areas to avoid emergence of traffic congestion, we propose a mechanism in which commuters bid their value of time and they can exchange funds with each other via dedicated auctions.

3. Model formulation

In this section, we present a mathematical formulation of the model used in our simulations as well as an overview of the simulation algorithm. The model is comprised of the following two main elements. The first one is a road network represented in the form of a directed graph $G = (V, E)$, where V is the set of vertices and E is the set of edges of the graph (since it is a directed graph, it is a set of ordered pairs of vertices, that is, $E \subseteq V \times V$). The second ingredient is a set of agents, K, in which each element (agent) is assigned to a particular road section in each iteration of the model. It is assumed that the drivers commute from one point of the city (vertex $\mathcal{A}$; e.g., their home) to another destination point (vertex $\mathcal{B}$; e.g., their work place). During peak hours, commuters will usually populate

K	population of agents
k	commuting agent, $k \in K$
N	the total number of agents ($N = \|K\|$)
V	set of vertices (intersections)
E	set of edges (road sections)
s	route
d_e	length of section (edge) e
E_s	set of road sections that route s consists of
$\overline{t_e}$	mean time of traversing section (edge) e
n_e	number of agents at route / edge e
N_e	capacity of route / edge e
v	commuter's velocity
S	a set of possible routes between origin and destination considered by an agent
s'	least cost route to a destination point
s''	second least cost route to a destination point
C_k	cost of traversing road s by agent k
C_k^F	fuel cost component
C_k^T	time cost component
$C_k^*(s)$	cost borne by agent k on road s under the optimal distribution
$C_k^\dagger(s)$	cost borne by agent k on road s under the Nash Equilibrium
$\Delta C_k^{*\dagger}$	cost difference between optimal and Nash Equilibrium distribution for agent k
F	the marginal cost of traveling each meter (ie. cost of fuel)
$\mathcal{W}_k$	k-th agent's value of time
$\mathcal{W}_k^B$	k-th agent's declared (bid) value of time
$\boldsymbol{\mathcal{W}}^{\boldsymbol{B}}$	vector of agent's declared (bid) value of time $[\mathcal{W}_1^B, \ldots, \mathcal{W}_N^B]^T$
$\Delta t_k^{*\dagger}$	travel time difference between optimal and Nash Equilibrium distribution for agent k
$\mathbf{x}$	in the 2-route scenario, a vector of routes taken by agents where $x_k = 0$ means that an agent k travels the route s_0 and $x_k = 1$ means that she travels s_1
$\mathbf{x}^*$	in the 2-route scenario, the optimal cost-minimizing value of $\mathbf{x}$
p_k	net money received by agent k from the auction

Table 1
List of symbols used in the paper

the streets on the route to their destination point by selecting the shortest path. This approach corresponds to a real life scenario where commuters use GPS navigation systems together with online traffic information (e.g., , e.g., a an app in their mobile phone). It is relatively simple to determine the choice of path for each agent under the reference scenario, since effective algorithms for searching for shortest paths in a weighted graph (such as Dijkstra algorithm or Yen's K-shortest paths) are already known.

We will now independently introduce the three main ingredients of the model. For convenience, Table 1 provides a summary of the variables and terminology used in this paper. In order to simplify the description throughout the paper, we will consider a scenario where routes are being travelled by commuters using their personal vehicles. However, without loss of generality, one route can represent a situation where a commuter is using her own personal vehicle while the other could correspond to a situation where commuter uses a public transportation system.

3.1. Route definition

Let S_{AB} be the set of all possible paths (no cycle allowed) between vertices $\mathcal{A}$ and $\mathcal{B}$:

$$S_{\mathcal{AB}} = \bigcup_{n=2}^{\infty} \left\{ W^n = (w_1, \ldots, w_n) : w_1 = \mathcal{A} \wedge w_n = \mathcal{B} \wedge \forall_{\substack{i \in \mathbb{N} \\ 2 \le i \le n}} : \ \left(w_{i-1}, w_i\right) \in E \right\}. \tag{1}$$

For a given route $s \in S_{\mathcal{AB}}$, let $E_s \subseteq E$ be defined as follows:

$$E_s = \bigcup_{2 \le i \le |s|} \left\{ (w_{i-1}, w_i) \right\};$$

that is, E_s is a set of edges the route s consists of. Here, $|s|$ denotes the number of vertices on route s; note that $|E_s| = |s| - 1$.

For simplicity, further in the theoretical part of the text we consider a set of routes $S \subset S_{\mathcal{AB}}$ and we will not focus on the actual structure of the graph (V, E). However, the graph representation of the transportation system is significant when implementing the actual model. We are using it in the implementation of the numerical agents-simulation experiments in the Section 6.

3.2. Costs of traveling

We will use $C_k : S \to \mathbb{R}_+$ to denote an individual cost function of traversing path $s \in S$ by an agent $k \in K$. The total cost of a given path reflects expenses on both gas and/or electricity necessary to propel the vehicle and idiosyncratic cost of time that has been spent commuting. It seems reasonable to assume that the price of fuel is equal for each agent, although, the individual value of time may vary. Nonetheless, we may assume that the value of time can be described by some statistical distribution typical for the entire population. Indeed, some experimental research has been carried out aiming to investigate the value of time among a population of people, e.g., [16]'s value pricing experiment.

The total cost of route s for agent k is then:

$$C_k(s) = C^F(s) + C_k^T(s),$$

where C^F is the fuel cost component and C_k^T is the time cost component for a given agent k. Furthermore, we can disassemble the fuel cost component as follows:

$$C^F(s) = \sum_{e \in E_s} d_e \cdot F,$$

where d_e is a length of road (measured in meters) represented by edge e, and F is the marginal cost of traveling each meter (ie. cost of fuel). Note that we assume here that each agent has the same marginal travel costs and hence the travel costs simply linearly depend on the distance travelled. On the other hand, the time component can be calculated as follows:

$$C_k^T(s) = \sum_{e \in E_s} \overline{t_e} \cdot \mathcal{W}_k,$$

where $\mathcal{W}_k$ is the k-th's agent value of time (measured in \$/min) and $\overline{t_e}$ is the mean time of minutes that it takes to travel between vertices connected by edge e.

For the purpose of traffic modelling, the following linear approximation of the velocity at each given road section is commonly used [8]:

$$v_e = (v_{\max} - v_{\min}) \times \left(1 - \frac{n_e}{N_e}\right) + v_{\min}. \tag{2}$$

As a result, the average time $\overline{t_e}$ can be easily computed as follows:

$$\overline{t_e} = \frac{d_e}{(v_{\max} - v_{\min}) \times (1 - \frac{n_e}{N_e}) + v_{\min}}. \tag{3}$$

3.3. Preferred route

We assume that each agent is rational and prefers the least costly route. Let $s'_{\mathcal{AB}}(k)$ be the most preferred route between $\mathcal{A}$ and $\mathcal{B}$ by the agent k. In order to increase readability whenever there is only one agent and one pair of points considered, we will denote the most preferred route simply by s'. An agent minimizes her cost function and hence:

$$s' \in S_{\mathcal{AB}} \quad \wedge \quad \forall_{s \in S_{\mathcal{AB}}} : \quad C_k(s') \leq C_k(s).$$

Moreover, we assume that traveling costs are different for each $s \in S_{\mathcal{AB}}$; in particular, it implies that s' is well defined. However, in practice, if there are more routes that minimize the cost, then we simply choose one of them at random.

Since we have provided each agent with the preferred route, we also need to specify at least one alternative route which the agent may reference to. The most natural selection of this alternative path is the second least-costly route. We denote the second-best route as s'' as follows:

$$s'' \in S_{\mathcal{AB}} \setminus \{s'\} \quad \wedge \quad \forall_{s \in S_{\mathcal{AB}} \setminus \{s'\}} : \quad C_k(s'') \leq C_k(s)$$

Again, by our assumption, s'' is well defined.

Normally, the agent will always choose the route s' over the route s'', unless she is offered a financial compensation $p_k(s')$ such as:

$$p_k(s') > C_k(s'') - C_k(s'), \tag{4}$$

that is, she receives more money than her cost of switching to the second-best route. As before, if there is more than one travel pattern satisfying the above condition, the agent chooses the second best route at random.

In the next section, we assume that each agent has only two alternative options: she can either stay on her current path, s', and offer other commuters a certain fee B to deviate from this path in order to allow her travel faster, or she can accept payments from the other commuters on this route and switch to her second-best route s''

4. Two route scenario

In order to check whether it is possible to improve the position of all system participants by allowing them to trade the road space between each other, we first consider the two-road model. This simplified variant is easy enough so that we are able to investigate the model theoretically.

We consider a single intersection $\mathcal{A}$ which splits into two different roads which then come together into the destination point $\mathcal{B}$. We will denote these roads as s_0 and, respectively, s_1, wherein we adopt a convention that in general road s_0 is "faster" than road s_1 (in terms of time necessary to traverse the road section when there are no other cars). The roads are described by the following parameters: d_s (length), N_s (capacity, the maximum number of agents that can occupy the road), and $v_{\max_s}$ (speed limitation).

Let us consider a situation in which N heterogeneous agents approach the intersection $\mathcal{A}$. Each agent has to decide which road she is going to take to reach point $\mathcal{B}$ (see Figure 1). The situation can be described by a vector $\mathbf{x}$ of zeros and ones, where $x_k = 0$ means that agent k travels the road s_0 and $x_k = 1$ means that she selects s_1. Let n_0 be the number of agents choosing road s_0 and $n_1 = N - n_0$ be the number of those who chose s_1. Let $\mathbf{x}_{-k}$ denote a vector of decisions of all agents other than agent k. Additionally, let $C_k(s_0|\mathbf{x}_{-k})$ be the total cost of agent k taking road s_0 and $C_k(s_1|\mathbf{x}_{-k})$ be the total cost of agent k taking s_1. For readability, we will later denote these values by $C_{0,k}$ and, respectively, $C_{1,k}$.

We will consider two particular cases. The first one is the **reference scenario** in which no additional incentive mechanisms are applied and the second one is the **optimal scenario** in which the overall cumulative utility of all agents is maximized by a central planner who receives information about heterogeneous values of time of each agent. By showing that the reference scenario outcome is in general not identical (that is, worse) to the optimal scenario, we provide an evidence that by introducing some mechanism allowing for better allocation of costs between its participants it is possible to improve utility of some agents (potentially all of them) while not worsening utility of the others.

4.1. Two-route model — reference scenario

In the reference scenario, each agent aims to optimize her own utility function (by minimizing her travel cost). We assume that agents make decision one-by-one and the equilibrium is achieved where there is no such agent who can improve their situation (this is similar to the "User Equilibrium" model discussed by [25]).

$$\forall_{k \in K : x_k = 0} \quad C_{0,k} \leq C_{1,k}$$

and

$$\forall_{k \in K : x_k = 1} \quad C_{0,k} \geq C_{1,k}.$$

In practice this means, if there existed an agent for whom the cost of her alternative road was lower, then she would clearly switch the road. Hence, the system converges to the Nash Equilibrium (NE) - no action of an individual agent can improve their situation.

This can be further written as:

$$\forall_{k\in K: x_k=0} \quad \mathcal{W}_k \cdot \overline{t_0}(\mathbf{x}) + d_0 \cdot F \le \mathcal{W}_k \cdot \overline{t_1}(\mathbf{x}) + d_1 \cdot F$$

$$\forall_{k\in K: x_k=1} \quad \mathcal{W}_k \cdot \overline{t_0}(\mathbf{x}) + d_0 \cdot F \ge \mathcal{W}_k \cdot \overline{t_1}(\mathbf{x}) + d_1 \cdot F,$$

In this equilibrium the drivers have chosen between roads s_0 and s_1. In order to simplify the exposition (the resulting formulas are much easier, while the qualitative conclusions remain the same), we assume, similar to the aforementioned [25], that the distance-related travel costs are similar on both roads. In our notation this assumption can be expressed as $d_0 \cdot F \simeq d_1 \cdot F$. This now leads to the following inequalities:

$$\forall_{k\in K: x_k=0} \quad \mathcal{W}_k \cdot \overline{t_0}(\mathbf{x}) \le \mathcal{W}_k \cdot \overline{t_1}(\mathbf{x})$$

$$\forall_{k\in K: x_k=1} \quad \mathcal{W}_k \cdot \overline{t_0}(\mathbf{x}) \ge \mathcal{W}_k \cdot \overline{t_1}(\mathbf{x}),$$

where $\overline{t_0}(\mathbf{x})$ and $\overline{t_1}(\mathbf{x})$ are times required for traversing the roads s_0. After eliminating the value of time by dividing both sides by $\mathcal{W}_k$, we get:

$$\forall_{k\in K: x_k=0} \quad \overline{t_0}(\mathbf{x}) \le \overline{t_1}(\mathbf{x})$$

$$\forall_{k\in K: x_k=1} \quad \overline{t_0}(\mathbf{x}) \ge \overline{t_1}(\mathbf{x}),$$

and so $\overline{t_0}(\mathbf{x}) \simeq \overline{t_1}(\mathbf{x})$ (since agents are discrete, these times are in general not equal but only approximately equal, however further we assume that the total number of agents is large and that at least a single agent is traveling each route). This means that the traverse time via both roads is similar once the Nash Equilibrium is reached. It is an intuitive result, since otherwise agents on the worse (slower) road would have switched to the other road and congestion (and traveling times) would level out in the end. For readability we will further denote $\overline{t_0}(\mathbf{x}) = \overline{t_0}$ and $\overline{t_1}(\mathbf{x}) = \overline{t_1}$. Additionally, in order to simplify formulas below and to get an intuition by investigating approximated values, we assume that v_{min} is close to zero but later in the numerical experiments in Section we fix $v_{min} = 10$ km/h.

Using simplified form of equation (3), where we assume that $v_{\min} \simeq 0$ and $n_s < N_s$ (that is at least one vehicle travels each road considered) we get:

$$\overline{t_s} \simeq \frac{d_s}{v_{\max_s} \cdot (1 - \frac{n_s}{N_s})}.$$

Considering that in our Nash Equilibrium the travel times are similar ($\overline{t_0} \simeq \overline{t_1}$) we have the following system of equations:

$$\begin{cases} v_{\max_0} \cdot (1 - \frac{n_0}{N_0}) \cdot d_1 \simeq v_{\max_1} \cdot (1 - \frac{n_1}{N_1}) \cdot d_0 \\ n_0 + n_1 = N, \end{cases}$$

where N_0 and N_1 are the maximum numbers of agents that can occupy the roads s_0 and, respectively, s_1 while n_0 and n_1 are the numbers of agents who are actually traveling the roads s_0 and s_1. Those equations can be easily solved for n_0:

$$n_0 \simeq \frac{N_0 \left(N d_0 v_{max_1} - N_1 d_0 v_{max_1} + N_1 d_1 v_{max_0} \right)}{N_0 d_0 v_{max_1} + N_1 d_1 v_{max_0}}$$

Moreover, if we assume that speed limitations for both roads are also similar (that is, $v_{\max_0} \simeq v_{\max_1}$) as well as the route length (that is, $d_0 \simeq d_1$), we get a solution for n_0 and n_1, for a scenario with two identical routes:

$$n_0 \simeq N \cdot \frac{N_0}{N_0 + N_1}, \quad n_1 \simeq N \cdot \frac{N_1}{N_0 + N_1}. \tag{5}$$

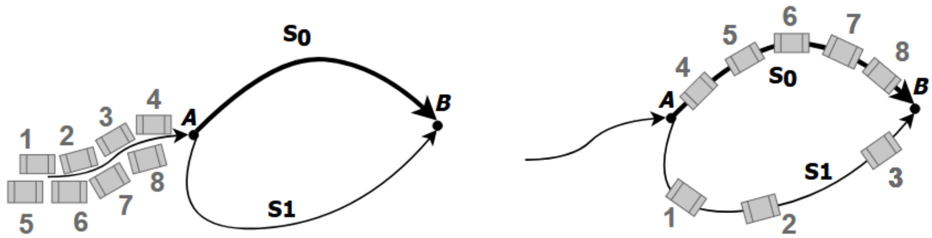

Figure 1: Example of possible agent arrangement in the reference scenario

(Note that both assumptions $v_{\max_0} \simeq v_{\max_1}$ and $d_0 \simeq d_1$ are only made here to make the formulas simpler; the equilibrium would be also obtained if we lifted them, similarly to what is discussed in [25].)

The above equation answers the question how the agents distribute over two roads which differ significantly only by their capacities (number of lanes), considering that Nash Equilibrium has been achieved. Figure 1 presents agents approaching an intersection and splitting into two roads in the reference scenario.

Taking the aforementioned results, we can estimate for each agent the cost borne by agent k on road s under the Nash Equilibrium (reference case) as follows:

$$\begin{aligned} C_k^{\dagger}(s) &\simeq F \cdot d_s + \mathcal{W}_k \cdot \frac{d_s}{v_{\max_s} \cdot \left(1 - \frac{N}{N_0+N_1}\right)} \\ &\simeq F \cdot d_s + \mathcal{W}_k \cdot t_{\min_s} \cdot \frac{N_0 + N_1}{N_0 + N_1 - N}, \end{aligned} \tag{6}$$

where $t_{\min_s} = \frac{d_s}{v_{\max_s}}$. Therefore, the total cost of travel borne by all agents in this scenario is:

$$C_{TOT}^{\dagger} = \sum_{k \in K} \left[C_k^{\dagger}(s_0) \cdot (1 - x_k) + C_k^{\dagger}(s_1) \cdot x_k \right] \tag{7}$$

Note that a crucial property of the Nash Equilibrium in our model is that it disregards agent's value of time because when arriving at the intersection, each agent chooses the route that is the least crowded at the time of making a decision. This means that there are $\binom{N}{n_1}$ pure strategy equilibria in this game. As it was mentioned before, since the agents are sequentially choosing between the routes, then each Nash Equilibrium leads to similar costs of travel for both routes s_0 and s_1, that is $C_k^{\dagger}(s_0) \simeq C_k^{\dagger}(s_1)$. Hence, in further discussion, we will assume (unless explicitly stated otherwise) that the total cost for an agent k does not depend on a particular established Nash Equilibrium; we will simply denote that cost as $C_k^{\dagger}$. Similarly, the total cost can be calculated separately for each Nash Equilibrium but, since our assumption is that $C^F(s_0) = C^F(s_1)$, we will simply express it by $C_{TOT}^{\dagger}$.

4.2. Two road model — optimal scenario

In this section, as opposed to the reference case, we look for a particular arrangement of all agents that maximizes their total utility of travel (or in other words minimizes total cost). By showing that the total utility of agents taking part in the simulation can be greater compared to the reference scenario, we can show that there is a potential for introducing a mechanism which incentivize some agents to take other road, impose the optimal agents arrangement, and then distribute the utility surplus among all its participants.

In order to find the optimal agents arrangement, an optimization model has to be introduced. The goal function of the problem is a total cost of travel of all agents and it is to be minimized:

$$C_{TOT}^{*} = \sum_{k \in K} \left[C_k(s_0) \cdot (1 - x_k) + C_k(s_1) \cdot x_k \right] \longrightarrow \min \tag{8}$$

The model is subject to the following restrictions:

$$\begin{aligned} &0 \le n_0 \le N_0 \\ &0 \le n_1 \le N_1, \end{aligned}$$

that is, the number of agents on roads s_0 and s_1 cannot exceed its maximal capacity, that is, N_0 and N_1, respectively. The next constraint

$$\sum_{k=1}^{N} x_k = n_1$$

binds binary vector x_k values with the total number of agent on road s_1. Finally, the constraint

$$n_0 + n_1 = N$$

makes sure that the sum of agents on both roads is equal to the number of agents approaching the intersection. Please note that the identification of the optimal scenario is a discrete optimization problem.

While this optimization allocation problem can be solved by a solver (such as Gurobi or Simplex), we have also identified a more computationally efficient Algorithm 1. The Algorithm 1 starts by sorting all agents by their value of time $\mathcal{W}_k$ where the agents having highest value of time will be placed in a better route and agents having lower value of time will be placed in the worse route — this is the starting step well known in the literature e.g. [29, 2]. Since the agents are sorted, at each step of the main loop a different split of agent allocation is considered. Note that we consider that each of the available routes can be "best". This is achieved in practice in the internal loop — depending on the value of $\mathbf{x}$ either s_0 is "good" and road s_1 is 'bad' or the situation is inverted — the s_0 is for the agents having low value of $\mathcal{W}_k$ and the route s_0 for agents having high value of $\mathcal{W}_k$. This dual approach ensures that the algorithm properly defines which road is better for people having high value of time with respect to income distribution and route capacity. It can be seen that when assuming a fixed consumption of fuel for each vehicle the global optimum solution must be included in the scenarios considered in Algorithm 1. Hence, the proposed approach can be used to find the minimum of the function (8) with the computational cost proportional to the number of agents. Note that the Algorithm 1 has been implemented in the Julia programming language and is available at the GitHub repository accompanying this paper[1] — see the `solve_travel` function. Moreover, we have also bench-marked this algorithm against a Solver-based implementation and discovered that it yields identical results while being 250 times faster (the source code is available in the `solve_travel_jump` function in the project online repository).

Once x^* (the optimal arrangement of agents) is found, for each agent we can calculate her change in travel costs:

$$\Delta C_k^{*\dagger} = C_k^* - C_k^\dagger. \tag{9}$$

Since, according to Algorithm 1, the actual layout of cars depends on value of time of agents $\mathcal{W}$ ($\mathcal{W} = [\mathcal{W}_1, \dots, \mathcal{W}_N]^T$) we will denote it in functional form as $C_k^{*\dagger}(\mathcal{W})$.

4.3. Toy scenario

We simulated a toy example in which we analyze a simple case where $N = 8$ agents approach an intersection $\mathcal{A}$ at which the road splits into two other roads, s_0 and s_1. Next, both roads connect at the intersection $\mathcal{B}$ which is a destination point for the agents. Although roads are of the same length, road s_0 has double capacity in comparison to s_1 ($N_0 = 20$ and $N_1 = 10$). The speed limitations are the same, namely, $v_{\max_0} = v_{\max_1} = 50$ km/h. The values of time for agents were chosen as follows: 1, 3, 5, 60, 80, 100, 150, and 200$/hr.

Next, we ran two simulations: the first one looking for the Nash Equilibrium and the other one searching for the global optimal distribution. Their outcomes have been compared with respect to each agent's cost of travel (see Equation 9). Table 2 contains key elements of both scenarios for all 8 agents.

The results show that the optimal distribution of agents is different from the reference one. One agent has switched her road from s_0 to s_1. The total cost for all agents has dropped by $0.42 from $32.68 to $32.26. At the end, some agents benefited while others lost with the new arrangement. However, after redistributing all costs and profits, slack funds remain that can be distributed among all participants. For the given optimal allocation of agents $\mathbf{x}^*$ we have shown a sample payment plan $\mathbf{p}$ that could be used to spread benefits of using the optimized routes $\mathbf{x}^*$ by all agents in the place of sub-optimal Nash Equilibrium $\mathbf{x}^\dagger$.

It has to be emphasized that the number of vehicles in our toy scenario is very small and so the travel times on both routes in the Nash Equilibrium are substantially different. In Table 2, among a few possible Nash Equilibria, we chose

[1] `https://github.com/jacfilip/RouteBidModel/`

Data:
$\mathcal{W}_k$ — value of time of each agent $k \in K$
C_0^F, C_1^F — cost of fuel used to travel roads s_0 and, respectively, s_1
$\overline{t_0}(n_0)$, $\overline{t_1}(n_1)$ — times of travel on roads s_0 and s_1 depending on the number of vehicles travelling that road
Result:
Optimal selection of roads $x_k \in \{0, 1\}$ for each agent k
Sort all agents by their offered bid values of time $\mathcal{W}_k$;
#Note that subsequent steps assume that the bidding agents have been ordered by their value of time
foreach $i \in \{0, 1, \dots, N\}$ **do**
    $\mathbf{x}^{(s_0)} = \begin{bmatrix} \mathbf{0}_{[i\times 1]} \\ \mathbf{1}_{[(N-i)\times 1]} \end{bmatrix}_{[N\times 1]}$ (place i of the agents with the highest value of time $\mathcal{W}_k$ at the road s_0)
    $\mathbf{x}^{(s_1)} = \begin{bmatrix} \mathbf{1}_{[(N-i)\times 1]} \\ \mathbf{0}_{[i\times 1]} \end{bmatrix}_{[N\times 1]}$ (place the remaining $(N-i)$ agents at the road s_1)
    $C^*_{TOT} = \infty$
    foreach $\mathbf{x} \in \{\mathbf{x}^{(s_0)}, \mathbf{x}^{(s_1)}\}$ **do**
        $n_0 = \sum_{x\in\mathbf{x}} 1 - x$;
        $n_1 = \sum_{x\in\mathbf{x}} x$;
        calculate travel times $\overline{t_0}(n_0)$ and $\overline{t_1}(n_1)$;
        $C = \sum_{k\in K} \left[C_k(s_0)\cdot(1-x_k) + C_k(s_1)\cdot x_k\right]$;
        if $C < C^*_{TOT}$ **then**
            $C^*_{TOT} = C$;
            $\mathbf{x}^* = \mathbf{x}$
        end
        ‘
    end
end
output: $\mathbf{x}^*$ reverted to the original sort ordering

Algorithm 1: Algorithm for finding the optimal travel allocation minimizing the total cost for travelers

Agent	$\mathcal{W}_k$	Reference (Nash Equil.)		Optimal scenario			$\Delta C_k^{*\dagger}$	Payment	Net profit
k	(\$/hr)	Route $\mathbf{x}^\dagger$	$C_k^\dagger$ (\$)	Route $\mathbf{x}^*$	C_k^*(\$)	Switch	(\$)	p_k (\$)	$p_k - \Delta C_k^{*\dagger}$ (\$)
1	1	1	2.13	1	2.14	NO	0.01	0.0625	0.0525
2	3	1	2.18	1	2.20	NO	0.02	0.0725	0.0525
3	5	1	2.24	1	2.26	NO	0.02	0.0725	0.0525
4	60	0	3.69	1	4.07	YES	0.38	0.4325	0.0525
5	80	0	4.22	0	4.09	NO	-0.13	-0.0775	0.0525
6	100	0	4.75	0	4.59	NO	-0.16	-0.1075	0.0525
7	150	0	6.07	0	5.83	NO	-0.24	-0.1875	0.0525
8	200	0	7.40	0	7.08	NO	-0.32	-0.2675	0.0525
Sum	–	$s_0: 5, s_1: 3$	**32.68**	$s_0: 4, s_1: 4$	**32.26**	**1**	**-0.42**	**0.0**	**0.42**

Table 2
Toy scenario for 8 vehicles traveling two roads — the Nash Equilibrium vs. a theoretical desired optimum (note that the positive value of p_k means that the agent k is receiving the money and the negative value that she need to pay it) .

an arrangement in which agents with the highest value of time already select the faster route. Hence, the calculated profit from optimizing their choices sets the lower limit. In other words, the overall benefit from optimizing route selections in this example would exceed \$0.42, if any other Nash Equilibrium happen to establish, because the Nash Equilibrium arrangement which has been chosen here is actually already the closest to the optimal one. By calculating all $\binom{8}{3} = 56$ combinations of pure strategy Nash Equilibria in the above example, we calculated that an expected (mean) benefit from auctions would be c.a \$0.82 and the maximal at \$1.24.

Figure 2 shows an example of how agents switch their road allocations in the optimal scenario. The presented result assumes perfect information and a central planner allocating agents to roads. However, note that if instead an auction mechanism were introduced to give incentives for agents to choose the optimal scenario, then it should be possible to achieve an improvement over a reference scenario. In particular, one agent, who in the reference scenario would rather choose road s_0, accepts a payment and agrees to choose a slower road s_1. Other agents who stayed on the road s_1 would also get paid, as their travel time extends after one agent joins them, thus their road becomes even

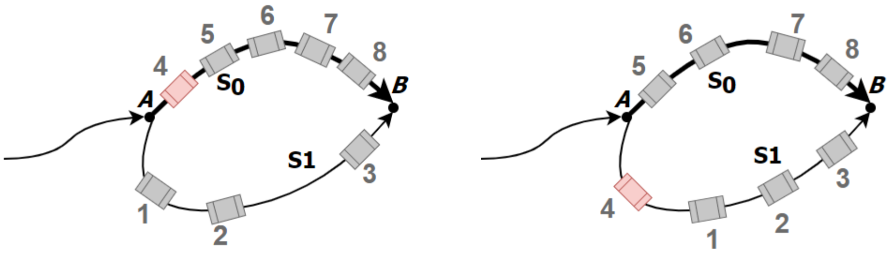

Figure 2: Example of agents arrangement in the optimal scenario. One of the agents (emphasised red), who in Nash Equilibrium scenario would have taken the road S_0, plans to take road S_1 under the optimal scenario.

more congested. On the other hand, all agents who stayed at s_0 gain, since their road became less congested, so they can share a part of their profit in the form of monetary incentive with agents travelling on s_1.

Exemplary payments p_k (where a positive sign means that an agent receives money) have been collated in Table 2, followed by the net incomes defined as a difference between p_k and cost difference of switching the road. Here we assumed that the total surplus of the transaction has been equally redistributed among all agents. As each agent has noted a positive net income (in terms of her utility), we can conclude that in the above toy example all agents gained by participating to this transaction.

To conclude, we have shown that under normal conditions — that is, when each agent independently chooses a faster route — the final agents' distribution is generally sub-optimal. By allowing heterogeneous agents to exchange funds in return for occupied road space, there is a possibility to create transactions at which all contractors increase their utility by reducing the total cost (with mutual payments balance included) of their travel. This approach would require a dedicated trading system to match buyers and sellers of the road space. Such a system, in the form of an auction, is the topic of the next section.

5. Auction mechanism implementation

5.1. Auction properties

In the previous section, we showed that it is possible to achieve a more beneficial distribution of agents on the roads. The question then emerges: how to incentivize commuters to choose more efficient travelling patterns. One of the solutions might be to establish a central planner that would charge commuters for choosing less congested roads. This approach was presented in the previous section and has a drawback, namely, it assumes that the planner would have a perfect information about the value of time of all agents involved in the trade. Therefore, in this paper we concentrate on an alternative solution where there is no supervisory unit; instead, we let agents exchange funds between each other via dedicated double auctions for road space.

Each double auction design can be characterized by some of the following attributes. Below we list those attributes and provide their brief explanation:

1. Individual Rationality (IR) – no agent loses by joining the auction;
2. Budget Balance (BB) – the total amount of purchase offers is equal to the amount of sales offers;
3. Truthfulness (TF) – auction mechanism gives no incentive either for under- or over-bidding; each bidder should reveal her true value of the good to maximize her expected reward; and
4. Economic Efficiency (EE) – the auction's outcome maximizes overall utility of all its participants.

Despite the fact that these properties are desired, Myerson–Satterthwaite theorem shows that it is impossible to have all of them incorporated into a single mechanism [19]. From the point of view of the analyzed auctioning system, IR is a crucial property because, otherwise commuters would be protesting against the introduction of such a system. Hence, it is necessary to assure that none of the agents suffer losses from taking part in the auction. EE is the second most important property of a designed mechanism, as our goal is to maximize overall utility of the commuters. However, it seems reasonable to relax this constraint by looking for a mechanism which is at least converging to the optimal solution. In the real life situation imposing the mechanism to produce the optimal outcome every time might not be possible due to the complexity of the problem and limited access to the information. BB is a constraint fairly easy to implement, for example, by redistributing the potential auction surplus among all the auction participants. Alternatively, we may allow the payments to be unbalanced and, providing that the auction surplus is positive, let it be utilized to cover the system infrastructure expenses or provide profit to the owner of the system. Regarding TF of the proposed approach, note that the optimal allocation of agents requires minimizing the total costs, as presented in Equation (8). Solving the cost optimization model presented in Algorithm 1 requires information about the agents' value of time. Since this information is going to influence the layout of agents, there is a natural temptation to manipulate the declared value which, in turn, makes it impossible to have TF property. However, our goal is to have an auction system that, in the worst-case scenario, proposes commuter allocations that are not worse than values acquired in the Nash Equilibrium.

To summarize, our goal is to propose an auction mechanism that provides the individual rationality (IR), balances the budget (or has a surplus that can be redistributed) (BB), and is economically efficient (EE) in the sense that the overall situation of all system participants is better than without the auction mechanism.

5.2. Auction design for two-route scenario

5.2.1. Bidding mechanism implementation

Let us consider a bidding system in which each agent declares her value of her time and, on this basis system, computes optimal cost-minimizing route arrangement. Additionally, a theoretical Nash Equilibrium (see Equation (5)) along its costs (see Equation (6)) are calculated. Moreover, in this subsection we assume that: (a) the number of commuters is large, and (b) as specified at the end of Section 4.1, all travel times in all Nash Equilibria are equal and hence it is irrelevant which particular Nash Equilibrium we use as the reference scenario.

Assume that p_k is an auction transfer (money received) calculated individually for each agent. In order to satisfy the IR property of the auction, this value should be greater than a theoretical difference between the cost of travel in optimal and Nash Equilibrium arrangement, that is,

$$p_k \geq \Delta C_k^{*\dagger}(\mathcal{W}^B), \tag{10}$$

where $\mathcal{W}^B = [\mathcal{W}_1^B, \dots, \mathcal{W}_N^B]$ is a vector of revealed (bid) value of time for each agent. For readability we will skip the declared bid value $\mathcal{W}^B$ in the text. We hold to the convention that a positive value of p_k means that the agent k receives money whereas a negative value means she spends the money.

In practice, this means that an agent who saves time in the optimal route allocation $\mathbf{x}^*$ will need to pay for it no more than the value of how much she saves; on the other hand, the agent who takes a longer route will receive a compensation that is enough to pay for her additional time. Note that this approach means that $p_k - \Delta C_k^{*\dagger}(\mathcal{W}^B) \geq 0$, that is, the auction has the *IR* property since the agents do not pay more than the profit they get from participating in the auction.

Theorem 1. *It is always possible to find a vector of values $\mathbf{p} = [p_1, \dots, p_N]$ satisfying the condition given by Equation 10, that is:*

$$\forall_{\mathcal{W}^B} \exists_{\mathbf{p}} \qquad \forall_{k \in K} \quad p_k \geq \Delta C_k^{*\dagger}(\mathcal{W}^B) \wedge \sum_{k \in K} p_k = 0. \tag{11}$$

Proof. Let us apply the Algorithm 1 with $\mathcal{W}^B$ as the the bid values to find the optimal allocation of agents $\mathbf{x}^*$. Note that under the optimal allocation of agents, $\mathbf{x}^*$, the total costs incurred by agents can not be worse (higher) than the cost in the Nash Equilibrium and hence (for readability we skip $\mathcal{W}^B$ parameter):

$$\sum_{k \in K} \Delta C_k^{*\dagger} = \sum_{k \in K} C_k^* - \sum_{k \in K} C_k^\dagger \leq 0. \tag{12}$$

Note that $\Delta C_k^{*\dagger}$ represents change in costs and hence the more negative value it has, the more efficient allocation. Additionally, note that in the $\mathbf{x}^*$ route layout if there is an agent who has lower costs than she would in the Nash Equilibrium, then there must be at least one agent who has higher cost than NE. Hence, it is always possible to find such a set of values $p_1, \dots, p_N$ that satisfy the condition defined in the Equation 10 and simultaneously that $\sum_{k\in K} p_k = 0$. One possible set of the values for $\mathbf{p}$ is:

$$p_k = \Delta C_k^{*\dagger} - \frac{\sum_{k\in K} \Delta C_k^{*\dagger}}{N}. \tag{13}$$

Since $\sum_{k\in K} \Delta C_k^{*\dagger} \leq 0$ than $p_k \geq C_k^{*\dagger}$ for all $k \in K$ and, obviously, $\sum_{k\in K} \left(\Delta C_k^{*\dagger} - \sum_{k\in K} \Delta C_k^{*\dagger}/N\right) = 0$. □

This means that the proposed mechanism fulfill the *BB* property. In practice, it is possible to find a set of payments such as $\sum_{k\in K} p_k < 0$ (e.g. see Table 2), that is, for some auctions there might be a budget surplus that could be used to finance running of the auction system. Further in the text we assume that the BB is obtained by equally dividing profits from the auction by all system participants along the definition in Equation 13. Obviously, for the above value of p_k, $\sum_{k\in K} p_k = 0$ and hence the BB condition is satisfied. Note that in this scenario sharing the profit from introduction of the auction $\sum_{k\in K} \Delta C_k^{*\dagger}$ among all N participants will not affect their bidding decision since increasing the bid by a unit increases bidder's share in the profit by $1/N$-th of the unit.

Agents' remuneration is dependent on their declared time value $\mathcal{W}_k^B$ (bid). Hence, the agents are going to adjust their bids in such a way that they do not overpay by bidding too high and they end up on a undesired route. They will also avoid bidding too low since it decreases their compensation when traveling the slower route. Let us analyze all possible agents' bidding behaviours and their potential effects. The bidding system ties the values of payments with the revealed value of time $\mathcal{W}_k^B$ in the following way: agents with higher $\mathcal{W}_k^B$ are assigned to a less congested road, therefore they become buyers and pay the amount $-p_k$; so the higher their bid is the higher they are obliged to pay. Agents with lower $\mathcal{W}_k^B$ are assigned to a more congested road, therefore they become sellers and receive the amount p_k; since their remuneration is proportional to their bid, hence the lower their bid is the lower they are paid. Suppose that an agent places a bid $\mathcal{W}_k^B$ that is greater than her true value of time $\mathcal{W}_k$. In that case, if she chooses the "worse" route, she can ultimately end up being assigned to the "better" road and she would have to pay a fee higher than the value of time that she has saved by traveling the better route (the value of p_k would be calculated with regard to the revealed bid values $\boldsymbol{\mathcal{W}}^B$). If she typically rather chooses the "better" road in the first place but she overstated her bid, then she would stay on the better road, but also she would have to pay a higher fee due to a higher declared value of time. On the other hand, if she underbids $\mathcal{W}_k^B < \mathcal{W}_k$, then she risks ending up in the worse route and suffering higher cost of time spent on commuting; or alternatively she can stay on the worse route but receive lower compensation due to underbidding her revealed value of time. This shows that the auction has a partial *TF* property — if the agents manipulate the prices they risk overpaying for their travel.

However, it should be noted that if the full information was available and the agents would be allowed to optimize their bids in reaction to decisions by other agents, the system would eventually converge to the Nash Equilibrium state. This scenario is described in detail in Subsection 5.2.2.

The presented bidding mechanism also improves global utility, since the system assigns each agent to a particular road in order to achieve optimal distribution $\mathbf{x}^*$, depending on their revealed values of time $\boldsymbol{\mathcal{W}}^B$. Further, if all agents were truthful, the system would ensure optimal arrangement, thus it would be EE. Still, even if agents are not fully sincere, we may assume that a rational agent with higher value of time will bid higher to will end up on a faster route and an agent with a lower value of time will rather take a slower route. Hence, in the end the road space distribution should be economically efficient, even if the revealed values of time are not exactly matched with the actual ones.

Summarizing, we can see that the proposed auction mechanism may fulfill two out of four desired properties: it is budged balanced and it provides individual rationality. Besides, it also partially provides economical efficiency and it is truthful if agents are risk averting.

5.2.2. *Stability of the auction*

In this section, we consider a possible outcome of an exemplary two-road auction and its stability in terms of game theory. We will show a model where each agent places the best bid as a response to all other agents' bids, and thus we simulate agents' rational behavior during actual auctions. The algorithm works for several steps, hence after many iterations all bids should reflect establishment of a long term global equilibrium among all participants (note that this equilibrium is a different Nash Equilibrium than the reference scenario discussed in Section 4.1 where there was no auction mechanism).

We are running an agent-based model of the auction setup considering the following parameters:

Parameter	Symbol	Value
Number of agents	N	200
Capacity of routes	$N_0,\ N_1$	150, 100
Real cost of time — randomized ($/h)	$\mathcal{W}_k$	$\sim LN(3,1)$
Route length (km)	$d_0,\ d_1$	10, 10
Marginal travel cost	F	0
Route minimum speed (km/h)	$v_{\min_0},\ v_{\min_1}$	10, 10
Route maximum speed (km/h)	$v_{\max_0},\ v_{\max_1}$	60, 60

Table 3
Parameters for the agent-based simulation model for of the auction system.

The simulation starts with each agent bidding their true value $\mathcal{W}_k$. The vector of bids, $\boldsymbol{\mathcal{W}}^B$, applied in Algorithm 1 allows to calculate the vector $\mathbf{x}^*$ that can be used to split the participants of the auction. (Recall that we use $x_k = 0$ to denote the fact that the agent k is traveling the route s_0 and $x_k = 1$ means that the agent k uses the route s_1.) In other words, agents traveling road s_0 that have the following set of indices $I^*_{s_0} = \{ k \mid x^*_k = 0 \}$ and the agents traveling the route s_1 that have the indices $I^*_{s_1} = \{ k \mid x^*_k = 1 \}$ and $I^*_{s_0} \cup I^*_{s_1} = K$. For both subsets of agents $I^*_{s_0}$ and $I^*_{s_1}$ their cardinalities are $|I^*_{s_0}| = N - \sum_{k \in K} x_k = n^*_0$ and $|I^*_{s_1}| = \sum_{k \in K} x_k = n^*_1$.

Analogously, we define the number of agents traveling routes s_0 and s_1 in the Nash Equilibrium as $n^\dagger_0$ and, respectively, $n^\dagger_1$. For each allocation of agents (n^*_0, n^*_1) and $(n^\dagger_0, n^\dagger_1)$, we can calculate (using Equation (3)) the corresponding times of travel $(t^*_{s_0}, t^*_{s_1})$ and, respectively, $(t^\dagger_{s_0}, t^\dagger_{s_1})$. Note that, following the discussion in Section 4.1, in all Nash Equilibria the times of travel are asymptotically equal and hence $t^\dagger_{s_0} = t^\dagger_{s_1} = t^\dagger$ and the values of $t^\dagger$, $n^\dagger_0$ and $n^\dagger_1$ do not depend on choice of a particular NE.

In order to analyze the stability of the auction system, we have set up the following scenario. In each step of the simulation, a single agent k observes bids of all other participants $\boldsymbol{\mathcal{W}}^B$ and finds a value $\mathcal{W}_k$ which optimizes her anticipating profit from participating in the auction: $p_k - \Delta C^{*\dagger}_k$. We test how this behavior of agents affects the global equilibrium. Note that in Algorithm 1 the agents are being sorted by their bid value. Since the allocation of routes is based on the obtained order, the most important information for agent is what is the bid value that changes her route. Let us denote that value by β and consider a scenario when an agent is travelling a slower route s_1. In order to change her location in the order obtained in Algorithm 1, she needs to bid at least $\beta = \min \{ \mathcal{W}_k \mid k \in I^*_{s_0} \} + \epsilon$ for some $\epsilon > 0$. On the other hand, we might have an agent on the route s_0 who might want to try to pay less — she might notice that a reduction of her bid might still leave her on a better route. However, assigning a smaller budget might also lead to a smaller number of cars traveling s_0, since this value is calculated in the process of solving the optimization problem. Finally, note that in computational scenarios we might have a situation that after observing all bids $\boldsymbol{\mathcal{W}}^B$, an agent decides to bid an amount equal to other participants. Whenever two or more bid values are equal, we give precedence to agents that bid more in the initial round where each agent has declared a unique value of their time.

In Figure 3 it can be seen that if actors had enough time to observe bids of our participants the system would eventually end up with a Nash Equilibrium which is less efficient. However, in any real world scenario the actors would not have full online information about bids of other system participants and they will not be able to update their bid several times while driving a car. Hence, when using the proposed system the agents will be in a more efficient situation than the Nash Equilibrium.

In conclusion, if agents are provided with enough information and are allowed to update their bids, then the efficiency of the transportation system will asymptotically approach to the Nash Equilibrium. In Figure 3, we show a sample dynamics of this process. Agents are updating their bids — one at each step of the simulation. The paying agents (buyers) will try to decrease their monetary contribution to the system while staying on the "better route". However, since this decreases the amount of money in the system it leads to an increase the number of cars on the fast route and decrease the number of cars on slow route. In turn, this leads to a situation where net costs of traveling both roads are increasing and the total costs of traveling (thick black lines) are approaching towards their level at the Nash Equilibrium (that is the 100% value in the presented figure). In this and subsequent figures, for better readability, we denote the vector of agents' costs in the optimal solution as $\mathbf{C}^* = [C^*_1, \ldots, C^*_N]^T$, the vector of money transfers

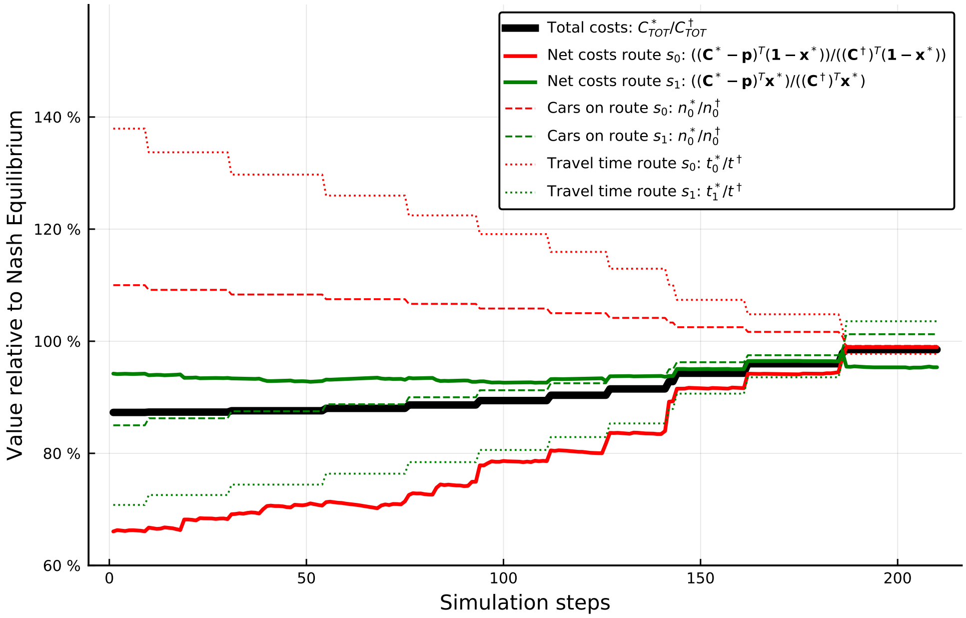

Figure 3: Sample system dynamics where full information about bids is available and agents can update them in reaction to other bids. In this scenario, the total cost (thick solid black line) steadily converges to a value slightly below the Nash Equilibrium.

as $\mathbf{p} = [p_1, \ldots, p_N]^T$, and the vector of agents' cost in the Nash Equilibrium as $\mathbf{C}^\dagger = [C_1^\dagger, \ldots, C_N^\dagger]^T$. Following the discussion in Section 4.1, we assume that the value of $\mathbf{C}^\dagger$ does not depend on the choice of a particular NE.

Note that in Figure 3, one can observe a switch of which road is "fast" around step 180. Such swaps of "fast" and "slow" roads can occur several times in response to update of bids by agents. Such change can occur even with each update of bids by agents — a sample dynamics of this type has been presented in Figure 4. Note that once the optimal load of roads is constantly changing, the total cost for all system participants is just gradually approaching the level of the Nash Equilibrium.

The analysis of the dynamics of the proposed bidding framework leads to the following conclusions:

1. in the worst-case scenario, the bidding system converges to a state equal to the Nash Equilibrium; and
2. in order to reach the worst-case scenario, the agents need to have full information about actions of other participants and need to get an opportunity to update their bids.

A practical implication of the first observation is that the commuter allocation will never be worse than the Nash Equilibrium where travel times naturally tend to be equal across all routes. Note that this can be easily proved. Consider a state of the system when one buyer is overpaying. If she had a full information, she would immediately lower her bid to match those of other sellers and when all bids are equal the allocation of drivers resulting from Algorithm 1 is identical to the Nash Equilibrium. Similarly, when an agent bids too low and it makes sense for her to bid higher in order to swap the road (or decrease the traffic on the current) — since the Nash Equilibrium is used as the baseline level for calculating the payments a rational increase of a bid will always increase the over-all efficiency of the system.

The second observation implies that in any practical real-life application the equilibrium offered by the bidding platform will be more efficient than the Nash equilibrium. In real life agents will not have full knowledge about other bids and it will not be feasible to update ones bid several times during a single auction. Note that when an agent does not know $\mathcal{W}^B$ at the time of placing her own bid, it is very likely that she may under- or over-estimate the value of β with an error greater than ϵ. Hence, when an agent becomes too greedy, it can result in redirecting that agent onto a less preferred route and in exacerbation of her utility. In this case it may occur that expected utility of placing bids too far from β may be unfavorable, due to the risk of landing on a less preferred route. This conclusion also means that the auction mechanism should not provide to participants the full information about bids so they can not adopt to the time values of other road network users.

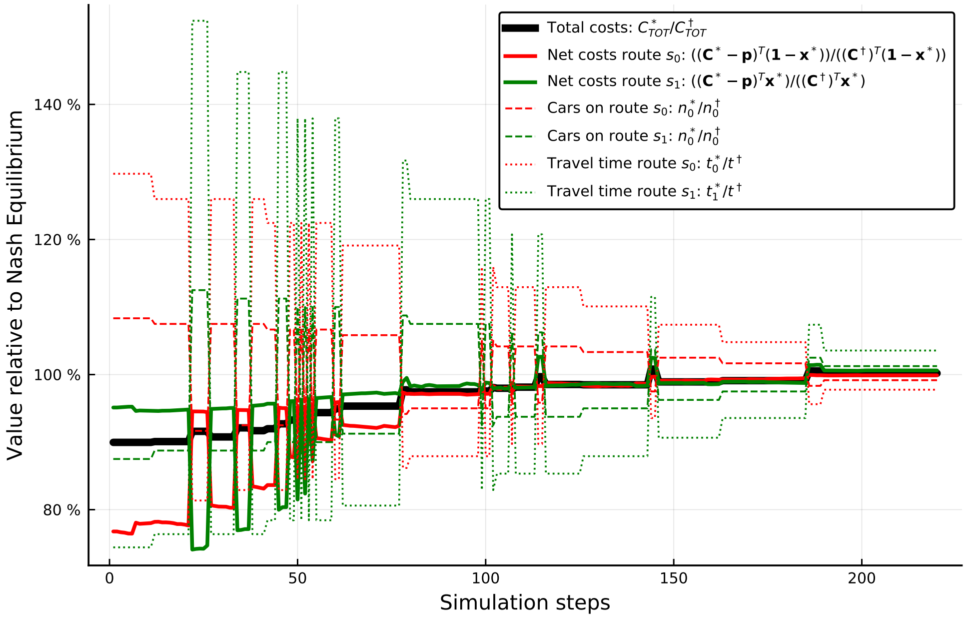

Figure 4: Sample system dynamics where full information about bids is available and agents can update them in reaction to other bids. In the process of updating bids the fastest route is changing and a bifurcation can be observed. However, the total cost (thick solid black line) steadily converges to the Nash Equilibrium.

5.2.3. Discussion on alternative auction designs

A natural question to ask is why each agent using the proposed system is actually paying a different price p_k. This seems to be the main incentive for price manipulation, that is, not declaring the real value of time $\mathcal{W}_k$ by agents participating in the auction. In order to illustrate it, let us consider a situation where instead of a vector of money transfers $p_1, \dots, p_N$ we have a single fixed price paid by all agents. In this scenario, we still need to use a vector of declared agent costs $\mathcal{W}^B$ because we would not be able to find the cost-optimal allocation of agents $\mathbf{x}^*$ otherwise. Let us assume that for a given optimal allocation of commuters $\mathbf{x}^*$, the road s_0 is "fast" and the route s_1 is "slow". In that case we can define the *middle* value of time as follows:

$$\overline{\mathcal{W}} = (\min\{\, \mathcal{W}_k \mid k \in I^*_{s_0} \,\} + \max\{\, \mathcal{W}_k \mid k \in I^*_{s_1} \,\})/2. \tag{14}$$

Note that all commuters who travel the "fast" route s_0 have their values of time higher than $\overline{\mathcal{W}}$ and all users who travel the "slow" route s_1 have their values of time lower than $\overline{\mathcal{W}}$. This allows us to calculate a fixed value profit from switching from route s_1 to route s_0 as (we assume that $C^F(s_0) = C^F(s_1)$):

$$\overline{\Delta C} = \overline{\mathcal{W}}(t_{s_1} - t_{s_0}). \tag{15}$$

This value can be used to calculate a fixed set of money transfers for agents traveling both routes:

$$\overline{p_k} = \begin{cases} -\frac{n_1^*}{N}\overline{\Delta C} & \text{for} \quad k \in I^*_{s_0} \\ \\ \frac{n_0^*}{N}\overline{\Delta C} & \text{for} \quad k \in I^*_{s_1} \end{cases}. \tag{16}$$

Note that $\sum_{k \in K} \overline{p_k} = 0$ and hence the proposed price calculation mechanism has the budget balance (BB) property.

Let us now consider the following alternative action design:

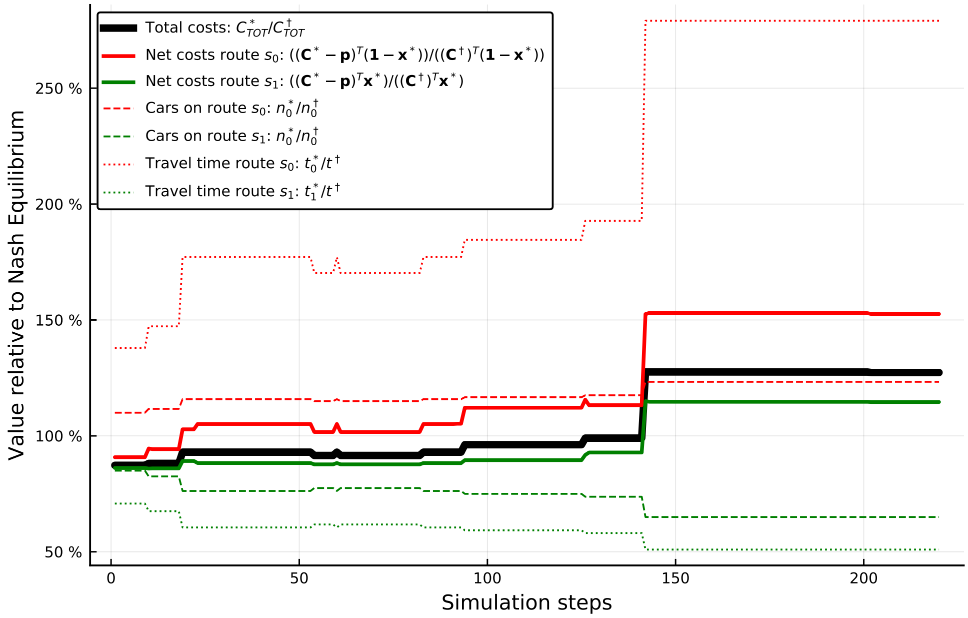

Figure 5: Typical dynamics for a scenario where a fixed price $\overline{p_k}$ is used rather then individual prices p_k depending on $\mathcal{W}^B_k$ declarations of agents. The system quickly diverges to a non-optimal state with total costs several times higher than those observed in the base Nash Equilibrium scenario.

1. Each agent declares her value of time $\mathcal{W}^B_k$.
2. Algorithm 1 is used to identify the optimal allocation of commuters.
3. The fixed profit from changing the route $\overline{\Delta C}$ is calculated using Equation (15).
4. The fixed money transfers $\overline{p_k}$ are calculated for each agent using Equation (16).

We have simulated the above mechanism design using assumptions presented in Table 3 and show results in Figure 5. It turns out that the fixed price auction design very quickly leads to an inefficient allocation of agents. In Figure 5, we have used exactly the same initial set of commuter time values $\mathcal{W}$ and the same optimal commuter allocation mechanism (Algorithm 1) as in Figure 3. Indeed, at time 0 all lines, except for *Net costs*, are starting at exactly the same locations on both figures. However, since the money transfers (p_k and $\overline{p_k}$) are calculated differently, users traveling the faster route have a strong incentive to overbid. On one hand, overbidding does not result in higher payments for an agent but, on the other hand, the overbidding pushes commuters away from the "better" route making more space for those who overbid (in Figure 5 the route s_1 is "better"). Note the green dotted and dashed lines, the number of cars using the route s_1, is constantly decreasing together with the travel time. Due to the non-linearity of Equation (3), the times on route s_0 are increasing much faster and thus generate far more costs than saving acquired on route s_1. Very quickly (around step 15) the money transfers received by agents traveling the "worse" route do no not cover extended travel times. Since the majority of agents incur losses, such system would not be accepted in a society where the payment mechanism is chosen by democratic voting. The next efficiency shift occurs at the step 142 where one agent places a very high bid that pushes several agents away from the faster route s_1 onto the slower route s_0. This also changes the middle value of time $\overline{\mathcal{W}}$ and drastically increases the difference in travel times between both routes, $(t_{s_0} - t_{s_1})$. As a result, the agents traveling the "better" route need to pay very high, yet insufficient, compensation to agents traveling the "slower" route. On average, both groups incur higher net costs in comparison to what they would incur in the Nash Equilibrium.

The presented discussion shows that auctions with a fixed "middle" price lead to outcomes that are inefficient for the majority of participants. This means that such mechanism design would not be accepted in a democratic society. Additionally, the overall average performance of the system naturally converges to a state when its total efficiency is far

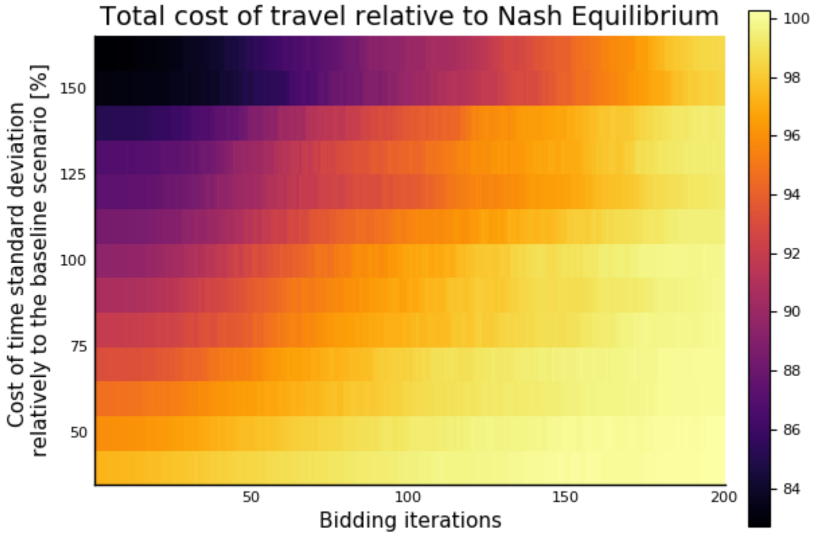

Figure 6: Heat map of total net costs of travel relatively to the Nash Equilibrium (lower value is better) as a function of agents' cost of time standard deviation and number of bidding iterations. It can be seen that the proposed bidding system is most efficient in societies with high variance of income distribution. However, even in societies with lower income variance, when agents do not have possibility to continuously update their bids the solution easily increases transportation system efficiency.

inferior to the efficiency that could be achieved in the Nash Equilibrium. Note that those are exactly the properties of the system that we have managed to avoid by proposing a variable price auction mechanism design in Subsection 5.2.1. The variable price auction mechanism design exhibits the worst-case performance equal to that in the Nash Equilibrium. Moreover, we have shown that it can be avoided by not providing full information to auction participants or not allowing them to immediately update their bids.

5.2.4. Sensitivity analysis of time value heterogeneity

The major assumption of our study is that the agents are heterogeneous across population — in particular, they have different value of their time. The important question is to understand how those cost-of-time disparities are actually affecting the efficiency of the systems and what is the dynamics of this efficiency with regards to information flow. Hence, we have conducted a sensitivity analysis aimed to present cumulative costs of travel as well as the rate of convergence to the Nash Equilibrium under different standard deviation of the real value of time $\mathcal{W}_k$. We follow the baseline scenario in which the assumption is that the cost of time is log-normally distributed with a distribution function parameters the same as in the example presented in Subsection 5.2.2 (that is, $\mathcal{W}_k \sim LN(3,1)$ which corresponds to the mean value of around 33.1 \$/h with the standard deviation of around 43.4 \$/h). We have simulated a total of 12 scenarios in which standard deviation was ranging from 40% to 160% of the baseline (with the step value of 10%). In each simulated scenario the agents were able to change their offers throughout 200 bidding iterations. Each of those 12 scenarios has been simulated 30 times with different randomization of the agents parameters and we present the mean values of results.

The heatmap presented in Figure 6 shows how the total net cost for all agents relatively to the costs under the Nash Equilibrium depends on the diversity of population's value of time and the number of consecutive bidding iterations (number of times the agents update their bids in response to actions of other system participants). The results gathered in Table 4 show that after a large enough number of iterations — regardless of the standard deviation of $\mathcal{W}_k$ — the cumulative net cost of travel is close to the cost received in the Nash Equilibrium scenario. However, along with the increase of diversity of the agents the net cost of travel starts to differ from the cost obtained in case where no bidding was allowed. The pattern emerges where the total cost of travel decreases as the standard deviation rises. The largest gap can be observed especially during the initial bidding iterations.

The sensitivity analysis results show that the bidding mechanism becomes more effective when the population is more diversified in terms of its value of time. It also suggests that it becomes even more efficient when its participants

$\mathcal{W}_k$ mean	$\mathcal{W}_k$ std. deviation		LN distribution parameters		Total costs $C^*_{TOT}/C^\dagger_{TOT}$ at iteration				
\$/h	\$/h	% of base	μ	σ	1	50	100	150	200
33.1	17.4	40	3.379	0.493	97.3	98.5	99.5	100.0	100.3
33.1	21.7	50	3.321	0.598	95.9	97.6	99.2	100.0	100.2
33.1	26.0	60	3.259	0.694	94.7	96.5	98.5	99.5	100.1
33.1	30.4	70	3.195	0.782	93.1	95.4	98.4	99.6	100.1
33.1	34.7	80	3.129	0.861	91.9	93.9	97.2	98.8	99.9
33.1	39.1	90	3.064	0.934	90.7	93.7	96.8	98.8	99.9
33.1	**43.4**	**100**	**3.000**	**1.000**	**89.4**	**92.7**	**96.7**	**99.0**	**99.9**
33.1	47.7	110	2.938	1.060	88.5	91.3	95.5	97.8	99.5
33.1	52.1	120	2.877	1.116	87.3	90.2	93.6	96.7	99.3
33.1	56.4	130	2.819	1.167	86.8	89.9	94.2	97.0	99.4
33.1	60.8	140	2.763	1.214	85.1	88.9	93.6	96.8	99.4
33.1	65.1	150	2.709	1.258	83.1	84.8	89.9	94.2	98.5
33.1	69.5	160	2.657	1.299	82.7	85.2	90.1	94.0	98.6

Table 4
Sensitivity analysis for total cost of travel relatively to the Nash Equilibrium calculated for different value of time standard deviations. The baseline scenario has been marked with bold font.

know less about each other preferences regarding travel time and thus about others bidding strategies.

6. Simulation of the auction system

In the previous section, we showed that the proposed mechanism can be effective on a small scale example. In this section, we test a road space bidding system at a large urban network environment. For this purpose, we designed and developed a computational framework dedicated to conduct multi-agent simulations of road traffic. The open-source framework written in Julia uses directed graphs to represent roads and intersections of actual locations. The full source code is available at GitHub[2]. A similar agent-based simulation based approach has been successfully used in various parts of our research program; see, for example, [11] or [10].

6.1. Numerical experiment assumptions

Each commuter is represented by an individual agent, equipped with unique properties and capable of making decisions regarding her current travel directions and bidding strategy. We make the following assumption for the simulation experiments:

1. The area for numerical experiments is southern Manhattan, New York City, USA. Vehicles are traveling north to the Manhattan island from Brooklyn via one of two bridges (Brooklyn bridge and Manhattan bridge) — see Figure 7.

2. Road lanes on the bridges are aggregated — all lanes within the bridge simultaneously suffer the same speed penalty from a given aggregated traffic.

3. The routes that do not constitute to the bridge do not suffer from congestion penalty. That is, the bidding mechanism is isolated to the bridge alone (that is, the through-output of all non-bridge roads is constant regardless of what traffic they handle).

4. Each agent has a defined starting point (randomly selected location in Brooklyn) and destination (a randomly selected location in Manhattan).

5. Each agent considers exactly two possible roads from South (Brooklyn) to the North (Manhattan) — one shortest road leading through eastern and one shortest leading through the western bridge (note that since agents have heterogeneous origins and destinations, each agent is evaluating a different pair of roads).

[2]https://github.com/jacfilip/RouteBidModel/

6. The simulation is stationary and reflects a single point of time of the highest congestion; thus the process of gradual development of the congestion is not analyzed.
7. Each agent has a heterogeneous value of her time.
8. At the first step (at the start of simulation), all agents place a bid on selecting either the eastern or the western road.
9. The actual travel time on each bridge depends on the number of agents that have chosen it. No congestion dynamics is modelled.
10. All decisions of agents regarding the chosen route and the bid value are taken before they start their trip and, once agents get to their car, they do not change them in reaction to the observed traffic.
11. Travelling through the bridge requires bidding the price and hence all agents are participants of the bidding system.
12. Similarly to the two-roads scenario, we assume that all agents place their bids reflecting their value of time and then the algorithm assigns each agent to a particular route so as to minimize the total cost of travel for all agents. Afterwards the algorithm calculates mutual payments between agents.

Imposing some of the aforementioned restrictions was necessary in order to keep the problem feasible to implement and solve computationally, and the outcomes to be clear and comprehensible. Note that the central part of the proposed bidding mechanism is the route allocation Algorithm 1 which time complexity is $O(n \log n)$ and hence the proposed approach can be applied to real life transportation systems. Nonetheless, we believe that all of these constraints are reasonable and do not produce a situation which could be considered as unlikely to happen in real life. Some of the assumptions could be easily dropped, although it could lead to obfuscation of the analysis. Especially, the assumption that there exist only two isolated roads (through the bridges) could be lifted, allowing the agents to select from many different routes leading to their destination. Alas, it would be incomparably more complex to assess quantitative impact of the auction on the agents in this case.

6.2. Results

Three different scenarios (configuration sets) of the simulation will be considered. In each scenario, 1,000 heterogeneous agents traversed from Brooklyn to Manhattan. Each scenario varied with assumed distribution of value of time among the population:

First scenario. The value of time has been drawn from a symmetrical triangular distribution with expected value of \$24 per hour, standard deviation of \$3 per hour, with a lower limit $a = \$16.65$, mode $m = \$24$, and an upper limit $b = \$31.35$.

Second scenario. Increased the degree of heterogeneity of the population are considered. The mean value of time has remained at \$24 per hour but the standard deviation raised to \$15. The parameters of the triangular distribution are $a = \$0$, $m = \$5.70$ and $b = \$66.30$.

Third scenario. A population with higher mean value of time (\$35 per hour) is considered. The standard deviation remains at a similar level to the the first scenario (\$3). Value of time was drawn from a symmetrical triangular distribution, with $a = \$27.65$, $m = \$35$, and $b = \$42.35$.

Figure 7 depicts cumulative paths of all agents in the simulation, where line hue depends on the intensity of traffic at the given road section, that is, red indicates the highest congestion, while blue indicates the lowest congestion. Table 5 shows aggregated results of aforementioned scenarios.

We calculated the total cost of travel for all agents in the Nash Equilibrium $C^{\dagger}_{TOT}$ (due to the large number of agents in NE, the choice of a particular NE is not important — see the discussion in Section 4.1) and in the optimal scenario C^{*}_{TOT}.

$$C^{*}_{TOT} = \sum_{k \in K} C^{*}_{k} \qquad \text{and} \qquad C^{\dagger}_{TOT} = \sum_{k \in K} C^{\dagger}_{k}$$

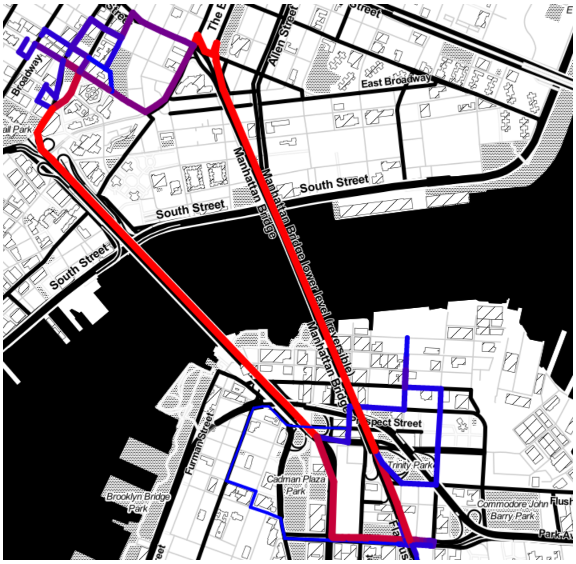

Figure 7: Routes taken by agents heading towards their destinations

Scenario	Agents choosing W/E bridge		Mean cost ($ p.c.)		Cost reduction
	Nash Equilibrium	Auctions	Nash Equilibrium	Auctions	
1	559/441	523/477	2.71	2.66	1.99%
2	562/438	482/518	2.69	2.53	5.84%
3	556/444	524/476	3.71	3.65	1.73%

Table 5
Aggregated results of road space bidding system simulation

Additionally, we denote the net cost of agent using the system as $(p_k - \Delta C_k^{*\dagger})$ (just to recall the positive value of p_k indicates that an agent has received money and negative one that she made payment).

The results show that, by introducing mutual payments between agents, it is viable to achieve an overall net cost reduction for all agents in each scenario. The average utility improvement (that is $(p_k - \Delta C_k^{*\dagger})/C_k^{\dagger}$) varied between 1.81% and 5.20% per agent. This value reflects an overall cost of travel decrease (that is, cost of fuel and cost of time, plus net auction payment). It is worth to mention that none of the agents increased their travelling cost after attending an auction in the optimal scenario, which shows that the bidding mechanism fulfilled individual rationality constraint.

By comparing the results of the first and the second scenario we conclude that the efficiency of the bidding system depends on a degree of heterogeneity of the commuters. By increasing standard deviation of their value of time, the relative improvement of the mean cost increased from 2.09% to 5.20%.

The results of the third scenario shows that increase of an absolute value of time does not affect the effectiveness of the auctioning system. Raising the mean value of time to $35/h while leaving the standard deviation on approximately the same level (that is, $3), resulted in a decrease of a relative cost saving from 2.09% to 1.81%. This occurred because the relative deviation of the value of time shrunk in comparison to the first scenario.

Figure 8 shows a distribution of bids among all commuters in each scenario. It can be easily noticed that all agents have been separated into two groups: buyers (ones with the negative values of p_k) and sellers (those with positive values of p_k). The results of the second scenario also shows that by increasing deviation of the cost of time, a total volume of auctions raises, as well as a spread between a minimal and a maximal bid.

Figure 9 presents a dependency between individual value of time of an agent and her auction net income in the second scenario (since in this scenario these correlations are the most visible), gathered from ten independent runs of this scenario in order to increase the sample. The picture shows that agents with lower value of time are the ones who are more likely to become sellers in the road space and, analogously, the agents with higher value of time are those who are more willing to pay for a possibility of faster travel. We can also notice that for both of these groups the absolute

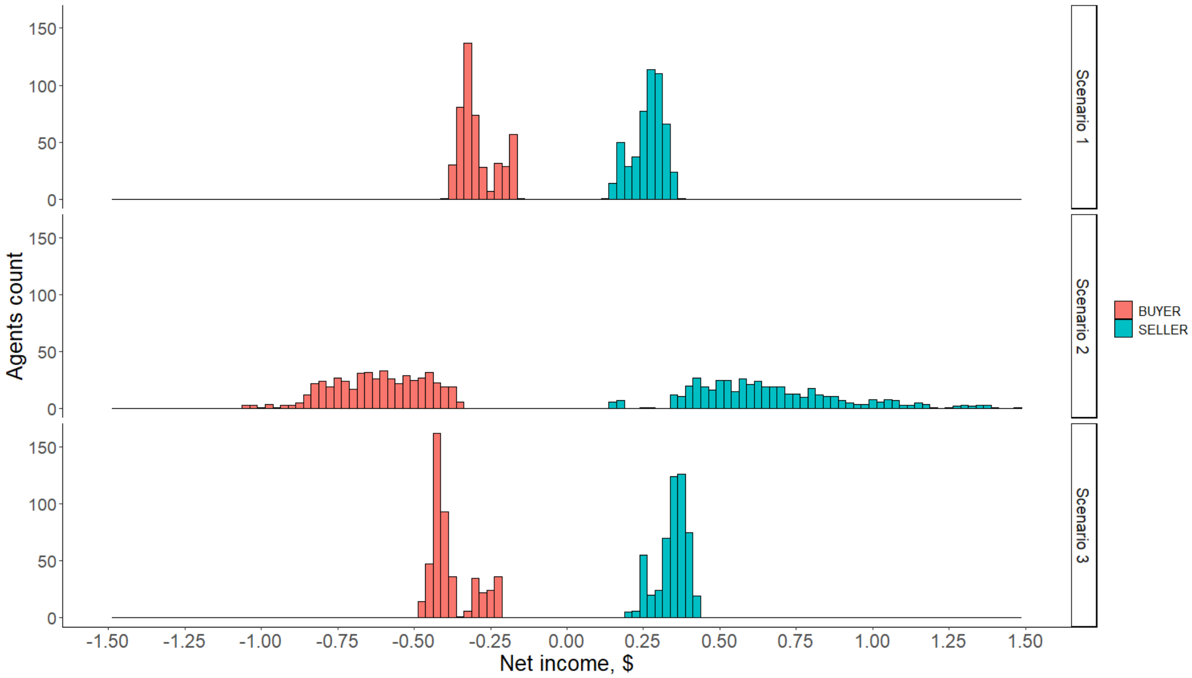

Figure 8: Histograms of auction payments in each scenario

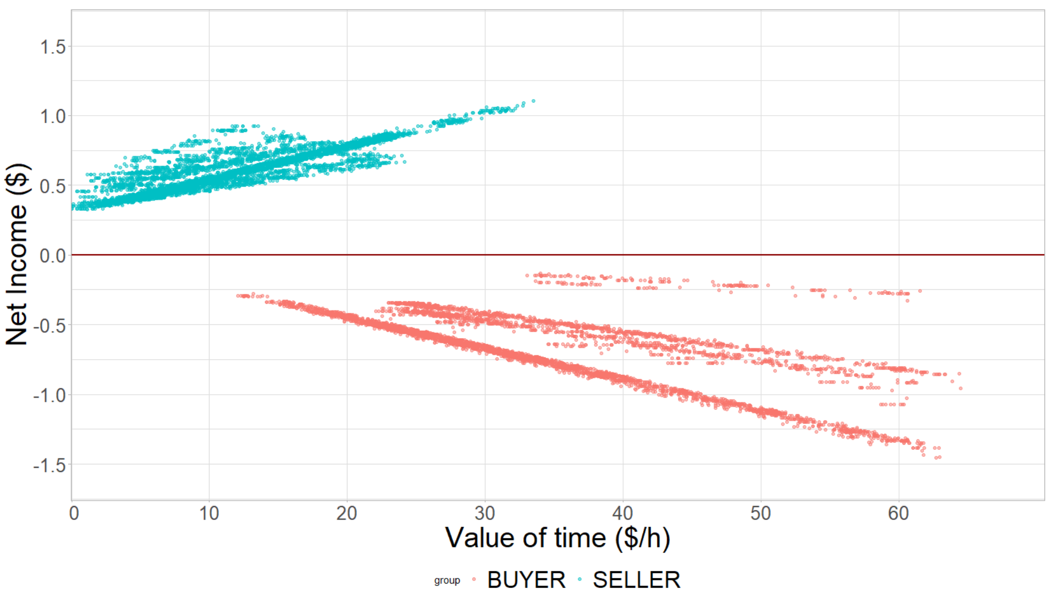

Figure 9: Levels of payments in function of value of time

value of payment increases with the cost of time. It shows that the buyers with higher evaluation of time are ready to pay more for the road space and the sellers with higher value of time require higher compensation for staying on the less favored route.

6.3. Discussion and practical implementation issues

While introducing a novel road pricing scheme may potentially be beneficial to all users of the transportation network, or even the entire local society, it also may create numerous practical issues and challenges, which have to be overcome prior to implementation. For example, [24] discuss practical concerns about privacy infringement caused by spatial price differentiation. Some other research on transportation systems resilience can be found in an article by [33].

One of the issues of the auction mechanism presented in our article may concern submitting false information regarding planned trips in order to receive remuneration for using the slower route (or mean of travel), despite having no intention to do so whatsoever.

In order to eliminate such undesired behavior, we need some kind of implementation protocol, which describes detailed rules for the presented auction mechanism. Below we present a set of rules that are required for the methodology presented in the paper which could actually be implemented in practice.

1. The system is used via an app running on a personal communication device, such as a mobile phone, and using the urban transportation network is only allowed when agent is using the application.
2. Each participant has to plan her trip ahead (e.g., at least 2 minutes earlier).
3. When planning a trip, participants must declare two points, start and destination, as well as a scheduled departure time. Each participant has to explicitly declare her intentions regarding her travel before she sets off.
4. Participant of the system places her bid (value of time, $\mathcal{W}_k^B$) for the particular travel.
5. On the basis of declaration the supervisory application uses Algorithm 1 to allocate the optimal route to an agent (note that this approach can be also used to distribute agents between public and private transportation).
6. Each user is allowed only to take route recommended by the application. However, if she wants change the path during the travel, she can place a new bid. A partially completed transaction would result in no income for sellers and a request of partial payment for buyers.
7. The system requires constant global positioning and tracking. This can be easily achieved by integrating the solution with GPS positioning systems ubiquitous in modern vehicles. For the users taking the public transportation it should be enough to check whether the travel actually took place.
8. Cash transfers are committed only when the travel is completed. This means that the bidding outcome would be considered as valid only if all above requirements have been met.

To summarize, clearly there are certain issues regarding practical implementation of a road bid system. However, as argued above, there are possible ways to overcome them. It is crucial to make the system resilient to overuses, since it determines its reliability and trustworthy of its potential provider. In practice, such system could be applied in a form of embedded feature of a vehicle or even as a mobile application. The users would be able to plan their trips (most likely they would mark their daily routes from from home to work and back), the hour of the beginning of the travel and then declared value of time in the form of bids. The system would find the optimal assignment of agents and calculate their payments basing on their bids and then it would communicate the results of the auction to the participants. Commuters would be obliged to follow the path suggested by the application in order to get paid. The operator of the application might be able to receive certain share of the auction surplus to cover her operation expenses and make desired profit.

Another issue is that we have simplified the way the travel times are calculated by just using the classic flow-based approach for computing travel times (see Equation 2). Classically, in the literature [13] models are divided into macroscopic (where global flows are calculated), microscopic (where individual behavior is modeled) and mesoscopic being a mix of the two previous approaches. In this paper we took the latter, mesoscopic approach. This allowed us to combine the micro-level (decision making of individual commuters) with a static flow modeling of congestion. The main advantage of the approach taken is it's computational efficiency combined with ability to fully model decision making situation of each participant. In turn this means that the proposed approach is computationally feasible for implementation in real world systems. Note that moving to a micro-model would mostly strengthen network effects in our simulation — this would further increase the incentive for bidders to participate in the auction mechanism.

In this article we focused on a practical use of such a system within urban networks, however it is worth noticing that this kind of infrastructure may also find its application on a larger scale. The same mechanism could be successfully applied to a highway system as an alternative (or a complement) to a traditional road tolling. Finally, as we pointed out earlier, the time complexity of the proposed optimization algorithm is $O(n \log n)$ which makes it possible to apply it in real-life scale.

7. Conclusions

In the paper we have presented an approach for improving (in terms of Pareto) the allocation efficiency of vehicle commuters in large and congested city centres, by introducing certain mechanisms allowing them to trade road space (and thus travelling time) in exchange for money.

First of all, we have analyzed a theoretical background of the presented problem by designing a simplified model in which a small group of heterogeneous agents could choose between one of the two different roads leading to their destination. In this simplified model, we considered two possible scenarios. In the first one, we assumed that the Nash Equilibrium establishes, that is, each agent chooses the lower cost road, knowing the choices of all other agents. This is the most common behavior among commuters in an every day traffic observation. In the second scenario, we searched for the optimal distribution of all agents, minimizing their overall cost of travel. We have presented scenarios where by creating certain incentives in terms of road selection, it is possible to improve situation of at least some of the agents, while not worsening utility of the others, that is, to improve global utility of all agents.

Afterwards, we have proposed a mechanism in which agents buy each other off via auctions in which they place bids representing their value of time and by these bids, the optimal arrangement is calculated along with the cash flows assigned to each participant. In order to find the optimal allocation of commuters, an algorithm with the computational efficiency of $O(n \log n)$ has been proposed. We have examined properties of this kind of auction such as individual rationality, truthfulness, budget balance and economical efficiency. We have also tested stability of these auctions by conducting a simulation in which agents consecutively place their best bids as a response to the bids off all other participants. We have shown examples where these kinds of auctions can be stable and have the worst-case outcome no worse than the Nash Equilibrium. However, normally when the users are not allowed to update their bids or full information on bids is not made available, the efficiency of the proposed design will be significantly superior to the efficiency that could be achieved in the Nash Equilibrium. Finally, in the paper it has also been shown that fixed-price auction designs cannot lead to stable and efficient allocation of commuters.

Next, we have transposed this auction mechanism into a large-scale simulation of agents travelling across a part of New York City. The outcomes of this multiagent simulations have confirmed the effectiveness of a described system. Hence, the proposed approach is a win-win solution for all participants of the transportation system. We also observed a dependency between a degree of heterogeneity of the population and the achievable unitary cost reduction.

Finally, we also tackled some practical issues which may occur in the real-life implementations of such systems. We discussed a number of possible ways of abusing such systems as well as appropriate solutions and prevention methods to combat them.

The proposed approach has many limitations that we plan to address in the future research. Firstly, in the paper we mostly focus on a scenario with 2-routes. I should be noted, however, that the our model is completely valid for other applications such as groups of people choosing between taking either public transportation or using their personal vehicles. The introduction of the proposed bidding algorithm could encourage more people to take the public transport and thus reduce the carbon-footprint of transportation networks. We plan to give more attention to that effect of our approach in our further research. Secondly, our model is using a static flow congestion approach rather rather then the queuing approach where the observed decision externalities are expected to be much stronger. Such dynamic models are a possible extension of this research for the future. Thirdly, in the model we assume that all the agents depart at the same time. An agent might incorporate into her decision problems the choice of departure time. In such scenario the decision problem would to choose in which auction the agent is participating in. We plan to explore the above problems in our future research.

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sights. NXM ensures data privacy and integrity by using a novel blockchain-based architecture which enables rapid and regulatory-compliant data monetization.

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