1st Graph Searching in Canada (GRASCan) Workshop

May 25–27, 2012
Ryerson University
Toronto, Canada

RYERSON UNIVERSITY
Everyone Makes a Mark

Friday, May 25

9:15 - 9:30	Opening remarks	
9:30 - 10:00	**Anthony Bonato**, Ryerson University *What is left to do on Cops and Robbers?*
10:00 - 10:30	**Gary MacGillivray**, University of Victoria *A simple method for computing the length of cop and robber games*
10:30 - 11:00	COFFEE
11:00 - 11:30	**Shannon Fitzpatrick**, University of Prince Edward Island *Cops and Robber on Circulant Graphs*
11:30 - 12:00	**Kieka Mynhardt**, University of Victoria *Protecting a Graph with Mobile Guards*

Saturday, May 26

9:00 - 9:30	**Andrew Beveridge**, Macalester College *The Petersen graph is the smallest 3-cop win graph*
9:30 - 10:00	**Nancy E. Clarke**, Acadia University *Ambush Cops and Robbers*
10:00 - 10:30	**Dieter Mitsche**, Ryerson University *Cops and invisible robbers: The cost of drunkenness*
10:30 - 11:00	COFFEE
11:00 - 11:30	**Abbas Mehrabian**, University of Waterloo *Cops and Robber Game with a Fast Robber on Interval Graphs and Chordal Graphs*
11:30 - 12:00	**Pawel Pralat**, Ryerson University *Revolutionaries and spies on random graphs*

Sunday, May 27

9:30 - 10:00	**Andrea Burgess**, Ryerson University *Cops and robbers on graphs associated with combinatorial designs*
10:00 - 10:30	**Margaret-Ellen Messinger**, Mount Allison University *Slow Firefighting*
10:30 - 11:00	COFFEE
11:00 - 11:30	**Stephen Finbow**, St. Francis Xavier *Discharging Fire-Works!*
11:30 - 12:00	**Boting Yang** (University of Regina) *Fast-Mixed Searching and Related Problems on Graphs*
12:00 - 12:30	**Gena Hahn** (University of Montreal) *Infinite cop-win graphs and some surprises*

Participants

Andrew Beveridge* (Macalester College)
Anthony Bonato* (Ryerson)
Andrea Burgess* (Ryerson)
Nancy Clarke* (Acadia)
Peter Danziger (Ryerson)
Danny Dyer (Memorial)
Stephen Finbow* (St. Francis Xavier)
Shannon Fitzpatrick* (PEI)
Ali Haidar (Ryerson)
Gena Hahn* (Montreal)
Bert Hartnell (St. Mary's)
Gary MacGillivray* (Victoria)
Abbas Mehrabian* (Waterloo)
Margaret-Ellen Messinger* (Mount Allison)
Dieter Mitsche* (Ryerson)
Kieka Mynhardt* (Victoria)
Pawel Pralat* (Ryerson)
Changping Wang (Ryerson)
Boting Yang* (Regina)
Vivija Ping You (Ryerson)

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Abstracts of Talks

Andrew Beveridge, Macalester College — Saturday 9:00 - 9:30
The Petersen graph is the smallest 3-cop win graph

In the game of *cops and robbers* on a graph $G = (V, E)$, k cops try to catch a robber. On the cop turn, each cop may move to a neighboring vertex or remain in place. On the robber's turn, he moves similarly. The cops win if there is some time at which a cop is at the same vertex as the robber. Otherwise, the robber wins. The minimum number of cops required to catch the robber is called the *cop number* of G, and is denoted $c(G)$. Let m_k be the minimum order of a connected graph satisfying $c(G) \geq k$. Recently, Baird and Bonato determined via computer search that $m_3 = 10$ and that this value is attained uniquely by the Petersen graph. We give a logical proof of this result. In the process, we prove some cop number results for n vertex graphs with maximum degree $\Delta(G) \geq n - 6$.

Coauthors: Paolo Codenotti (Institute for Mathematics and its Applications), Aaron Maurer (Carleton College), John McCauley (Haverford College), and Silviya Valeva (University of Iowa)

Anthony Bonato, Ryerson University — Friday 9:30 - 10:00
What is left to do on Cops and Robbers?

Despite much of the foundational work on Cops and Robbers games, there are a many unsolved problems and conjectures in the field. Meyniel's conjecture, giving an $O(\sqrt{n})$ upper bound on the cop number of a connected graph of order n, is a prominent example. We survey some of the central open problems on Cops and Robbers, and focus on a few natural variants of the game which are less well understood.

Andrea Burgess, Ryerson University — Sunday 9:30 - 10:00
Cops and robbers on graphs associated with combinatorial designs

Combinatorial designs and finite geometries have proved useful in the study of cops and robbers; incidence graphs of projective planes and (partial) affine planes have been found by Prałat and by Baird and Bonato, respectively, to yield Meyniel extremal families of graphs, i.e. families of graphs for which the cop number is at least $d\sqrt{n}$, where n is the order of the graph and d is a fixed constant. We examine the game of cops and robbers on various classes of graphs constructed from combinatorial designs and finite geometries, giving bounds on the cop number and the exact value in some instances.

Joint work with Anthony Bonato.

Nancy E. Clarke, Acadia University — Saturday 9:30 - 10:00
Ambush Cops and Robbers

A variation of the Cops and Robber game is introduced in which the robber side consists of two robbers. The cops win by moving onto the same vertex as *one* of the robbers after a finite number of moves. As in the original game, the robber side can win by avoiding capture indefinitely. In this version, however, they can also win by both moving onto the same vertex as the cop at the same time. Otherwise, the robbers must be located on distinct vertices. We present a variety of results including a recognition theorem for graphs on which a single cop can guarantee a win, as well as a strategy that can be used by the cop to win on such graphs.

Joint work with M. Creighton.

Stephen Finbow, St. Francis Xavier — Sunday 11:00 - 11:30
Discharging Fire-Works!

The firefighter problem models the spread of a fire on a graph or network while firefighters or defenders try to limit the extent of the damage to the network. The surviving rate, the expected percentage of the network saved, is arbitrarily close to zero for almost every graph. We will discuss how the discharging rate can be successfully employed to bound the surviving rate of a subclass of planar graphs away from zero.

Shannon Fitzpatrick, University of Prince Edward Island — Friday 11:00 - 11:30
Cops and Robber on Circulant Graphs

The game of Cops and Robber is played on a with the two sides alternating moves, and all players knowing the others' position at all times. The copnumber of a graph is the minimum number of cops required to guarantee the robber's capture.

A circulant graph $C_{n;m,k}$ is graph with vertices $x_0, x_1, \ldots, x_{n-1}$ and edges $x_i x_{i+m}$ and $x_i x_{i+k}$ for each $i = 0, 1, \ldots, n-1$, where addition is done modulo n. In this talk, we demonstrate that all connected circulant graphs $C_{n;m,k}$ have copnumber at most three, and $C_{n;m,k}$ has copnumber one only if it is complete. In addition, we present a number of conditions on n, m and k that result in a copnumber of at most two.

Joint work with J.P. Larkin, an undergraduate student at UPEI.

Gena Hahn, University of Montreal — Sunday 12:00 - 12:30

Infinite cop-win graphs and some surprises

A quick survey of infinite cop-win graphs underlying the rather different nature of the infinite as opposed to the finite. Some recent work in progress with Robert Woodrow shows a few surprising properties.

Gary MacGillivray, University of Victoria — Friday 10:00 - 10:30

A simple method for computing the length of cop and robber games

There are a variety of known methods for determining the length of a cop and robber game under the assumption of optimal play by both sides. Each of them uses some sort of auxiliary structure. We describe a method which uses only local information about the graph, and which is easy to carry out with pencil and paper.

Joint work with Nancy E. Clarke (Acadia University) and Stephen Finbow (St. Francis Xavier University).

Abbas Mehrabian, University of Waterloo — Saturday 11:00 - 11:30

Cops and Robber Game with a Fast Robber on Interval Graphs and Chordal Graphs

We consider a variant of the Cops and Robber game, introduced by Fomin, Golovach, Kratochvíl, in which the robber has unbounded speed, i.e. can take any path from her vertex in her turn, but she is not allowed to pass through a vertex occupied by a cop. We study this game on interval graphs and chordal graphs. Let $c_\infty(G)$ denote the number of cops needed to capture the robber in graph G in this variant. We show that if G is a connected interval graph, then $c_\infty(G) = O(\sqrt{|V(G)|})$, and that for every n there exists a connected n-vertex chordal graph G with $c_\infty(G) = \Omega(n/\log n)$.

Margaret-Ellen Messinger, Mount Allison University — Sunday 10:00 - 10:30

Slow Firefighting

In firefighting and graph searching problems, a contaminant typically spreads very quickly. We introduce a variant in which the contaminant spreads at a slower rate. At each time step, the searchers (or firefighters) protect one vertex and the contaminant (or fire) spreads from one vertex to all unprotected, unburned neighbours. On infinite graphs, the goal of the firefighter is to surround the burned vertices with a set of protected vertices, while the goal of the fire is to avoid such containment. We will consider results on both the two-dimensional and three-dimensional grids.

Dieter Mitsche, Ryerson University — Saturday 10:00 - 10:30

Cops and invisible robbers: The cost of drunkenness

We examine a version of the Cops and Robber game in which the robber is invisible (CiR), i.e., the cops do not know his location until they capture him. We examine two variants: in the first the robber is adversarial (he actively tries to avoid capture); in the second he is drunk (he performs a random walk). We study the invisible Cost of Drunkenness (iCOD), which is defined as the ratio $cti(G)/dcti(G)$, with $cti(G)$ and $dcti(G)$ being the expected capture times in the adversarial and drunk CiR variants, respectively. In this talk we provide bounds for d-regular trees and grids, and time permitting, we give general upper and lower bounds for general classes of graphs.

Joint work with A. Kehagias and P. Prałat.

Kieka Mynhardt, University of Victoria — Friday 11:30 - 12:00

Protecting a Graph with Mobile Guards

Graph protection involves the placement of mobile guards on the vertices of a graph to protect it against single or sequences of attacks. Here the focus is on securing the vertices and edges of graphs against infinite sequences of attacks, executed one at a time. At most one guard is stationed at each vertex, and guards do not return to their original positions before facing another attack. A number of different protection models have been studied. A survey of the latest results and open problems will be presented.

Pawel Pralat, Ryerson University — Saturday 11:30 - 12:00

Revolutionaries and spies on random graphs

A team of r revolutionaries tries to hold an unguarded meeting consisting of m revolutionaries. A team of s spies wants to prevent this forever. For given r and m, the minimum number of spies required to win on a graph G is the spy number $\sigma(G, r, m)$. The game was invented by József Beck in the mid-1990s and generated some interest recently. We present asymptotic results for the game played on random graphs $G(n, p)$ for a large range of $p = p(n)$, $r = r(n)$, and $m = m(n)$.

Joint work with D. Mitsche.

Boting Yang, University of Regina Sunday
Fast-Mixed Searching and Related Problems on Graphs 11:30 - 12:00

We introduce the fast-mixed search model, which is a combination of the fast search model and the mixed search model. Given a graph G in which a fugitive hides on vertices or along edges, the fast-mixed search problem is to find the minimum number of searchers to capture the fugitive in the fast-mixed search model. We establish relations between the fast-mixed search problem and other graph search problems. We also establish relations between the fast-mixed search problem and the induced-path cover problem. We present linear-time algorithms for computing the fast-mixed search number and optimal search strategies of some classes of graphs, including trees, cacti, and interval graphs. We prove that the fast-mixed search problem is NP-complete; and it remains NP-complete for graphs with maximum degree 4.