SLOW FIREFIGHTING

M.E. Messinger
S.P. Yarnell
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THE PYRO GAME

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Two-player game played on a connected graph

At step 0, the pyro burns vertex ν_0

At step $t > 0$, the firefighter "moves" and then the pyro "moves"

select a burned vertex ν and spread ***from*** ν to all unprotected, unburned neighbours of ν.

Goals?

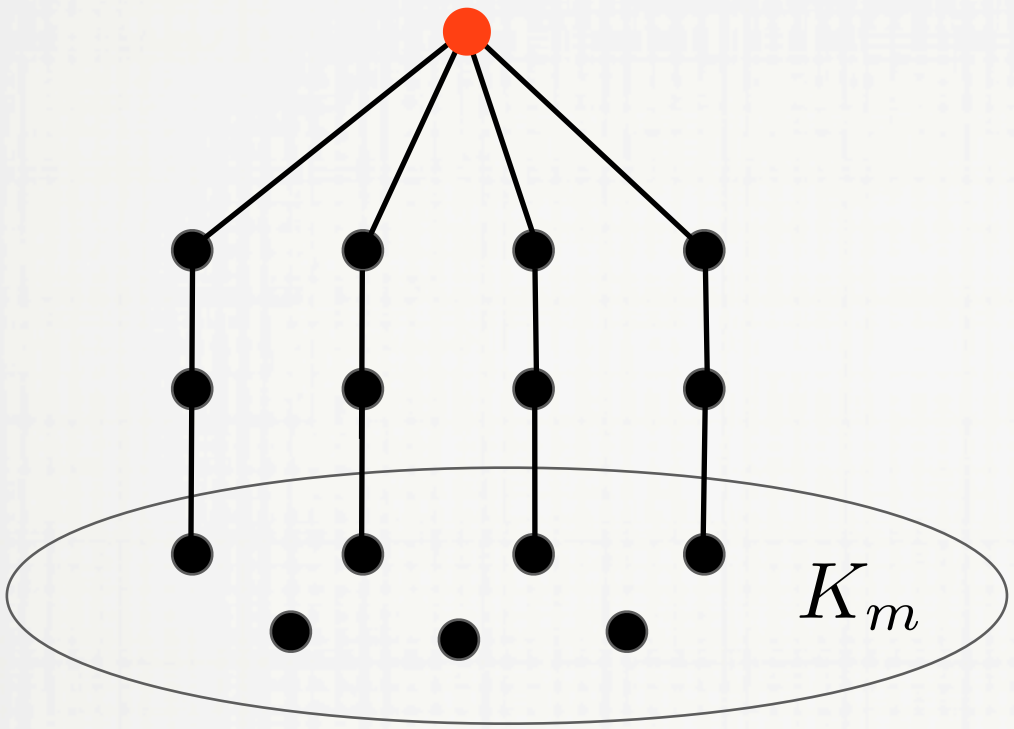
pyro

- maximize burned vertices?
- burn a particular set of vertices?
- avoid containment?

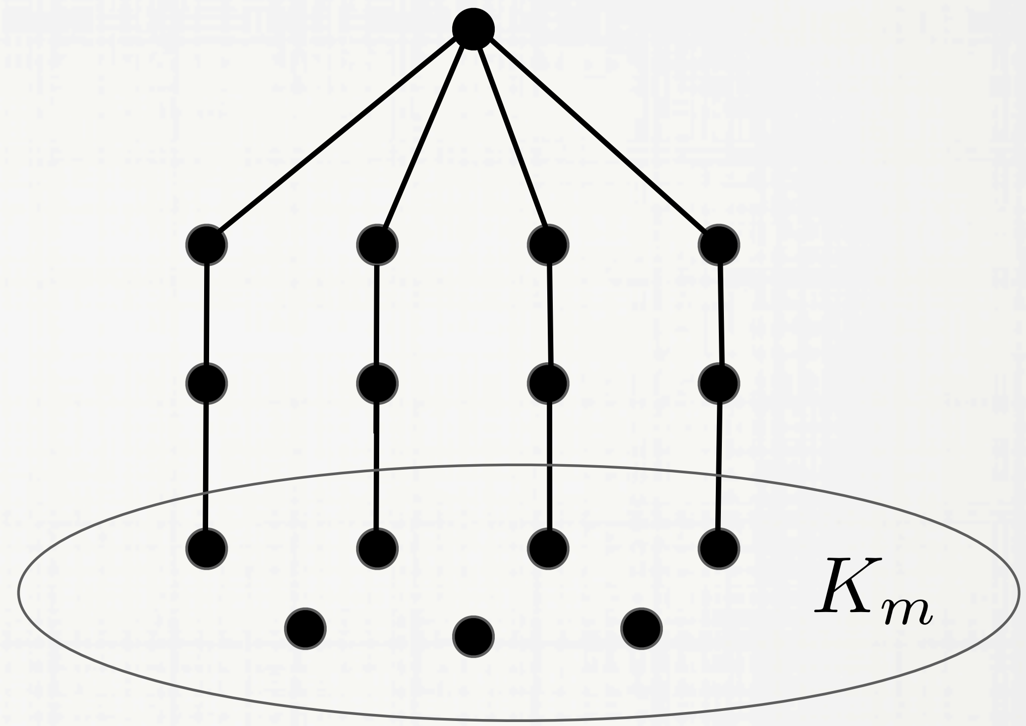
firefighter

- minimize burned vertices?
- save a particular set of vertices?
- contain the pyro?

The Firefighter Problem



The Pyro Game



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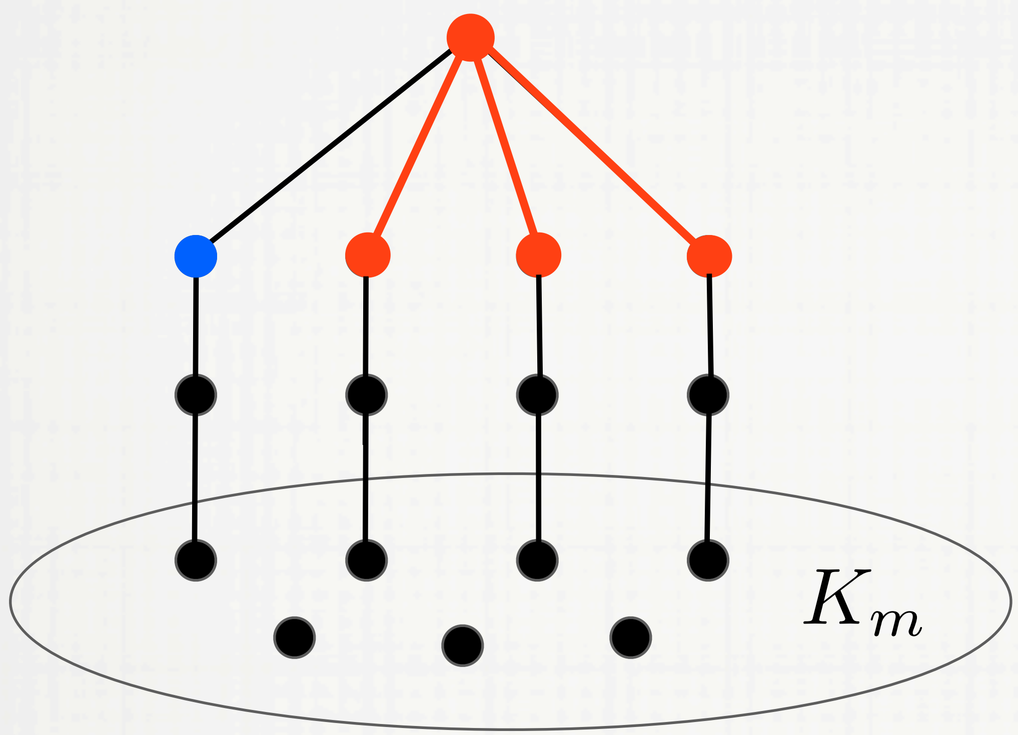

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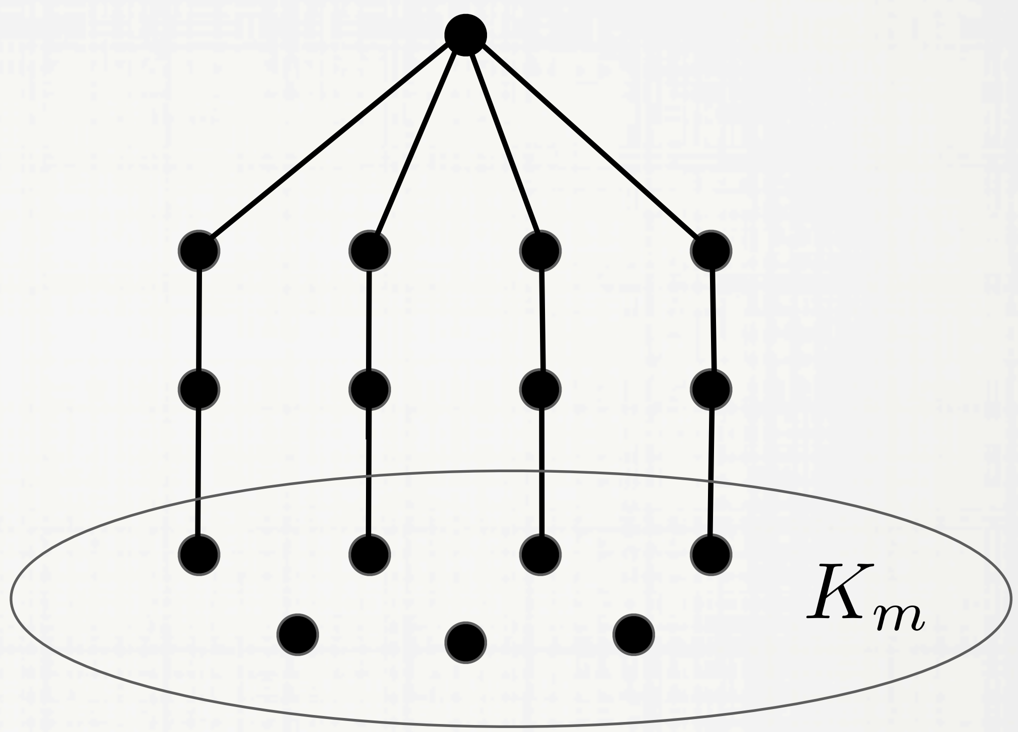

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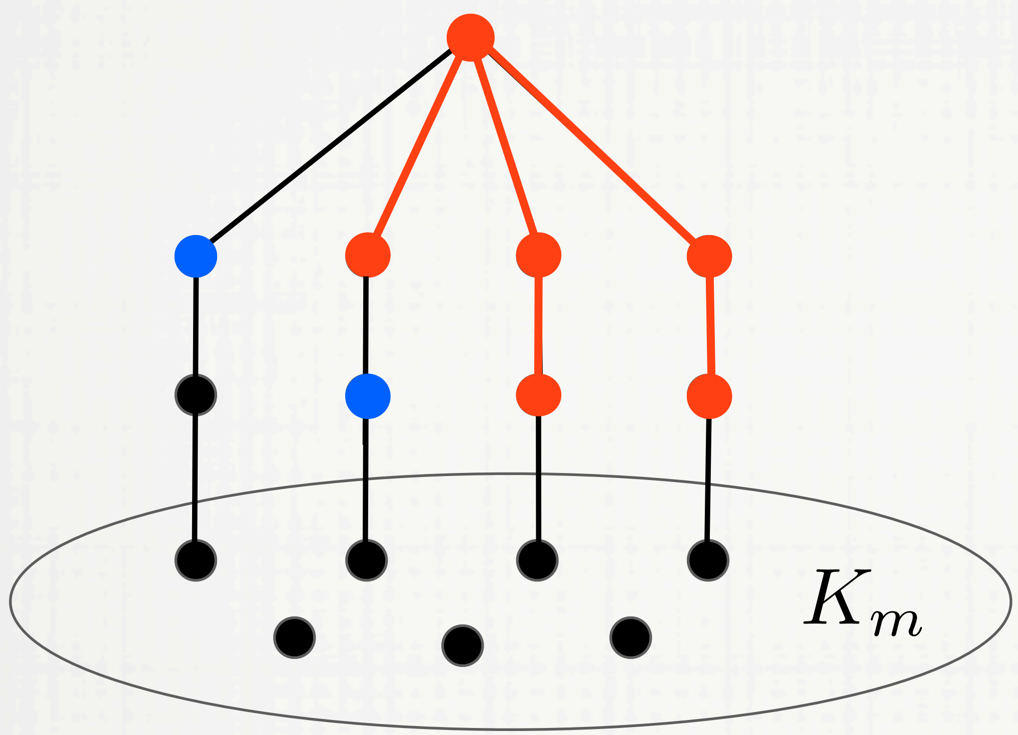

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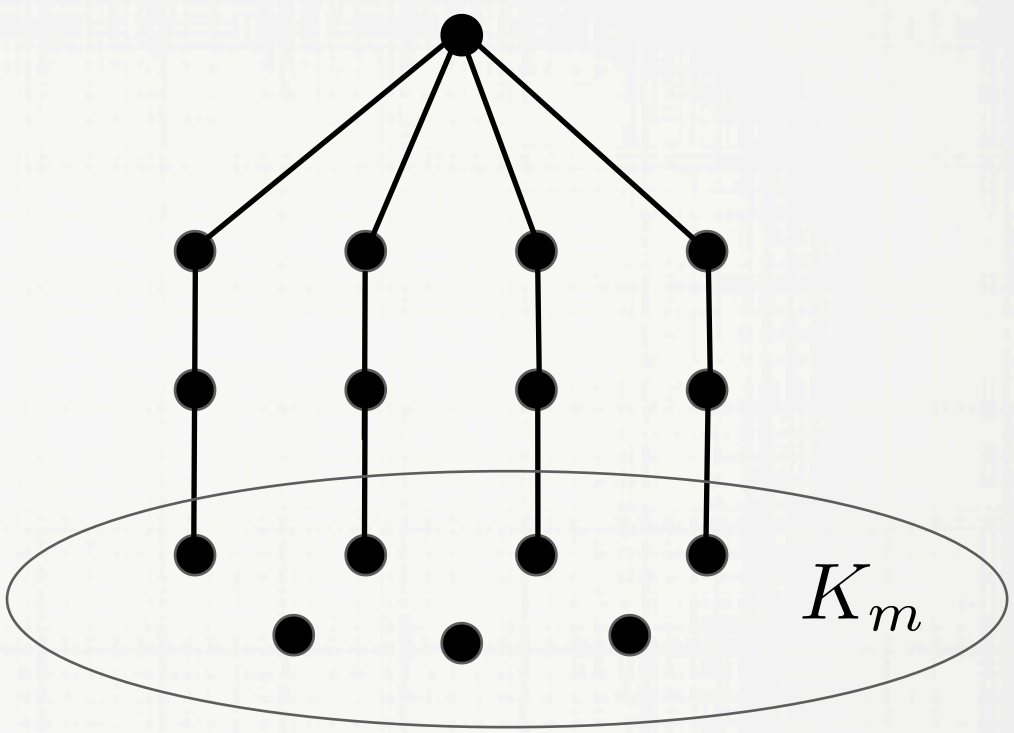

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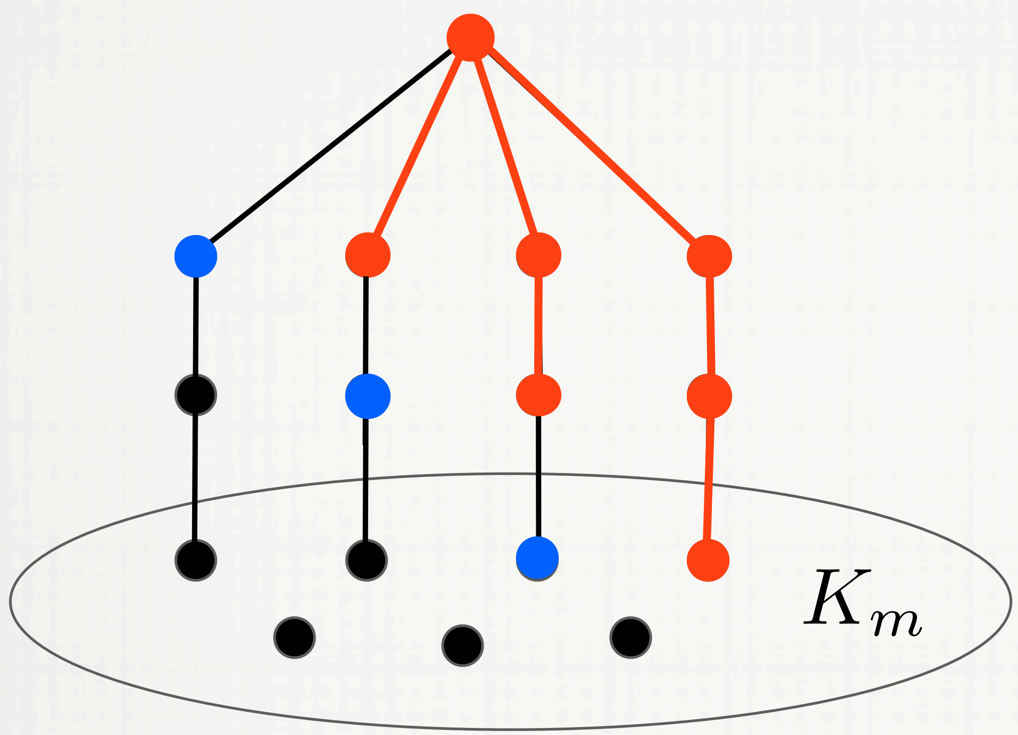

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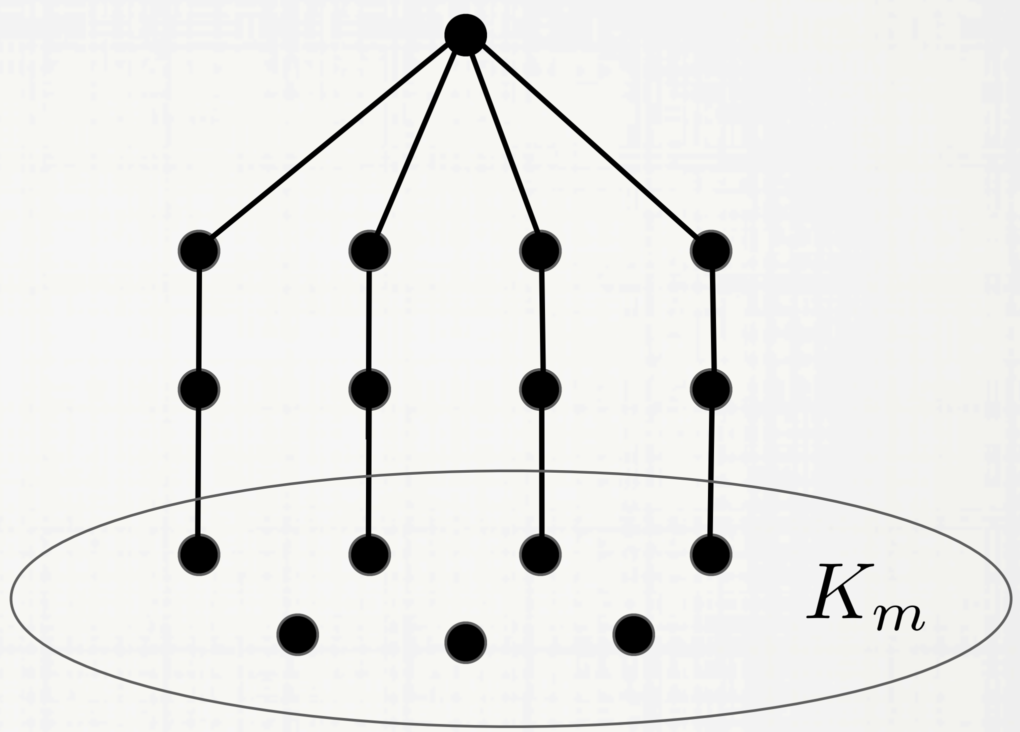

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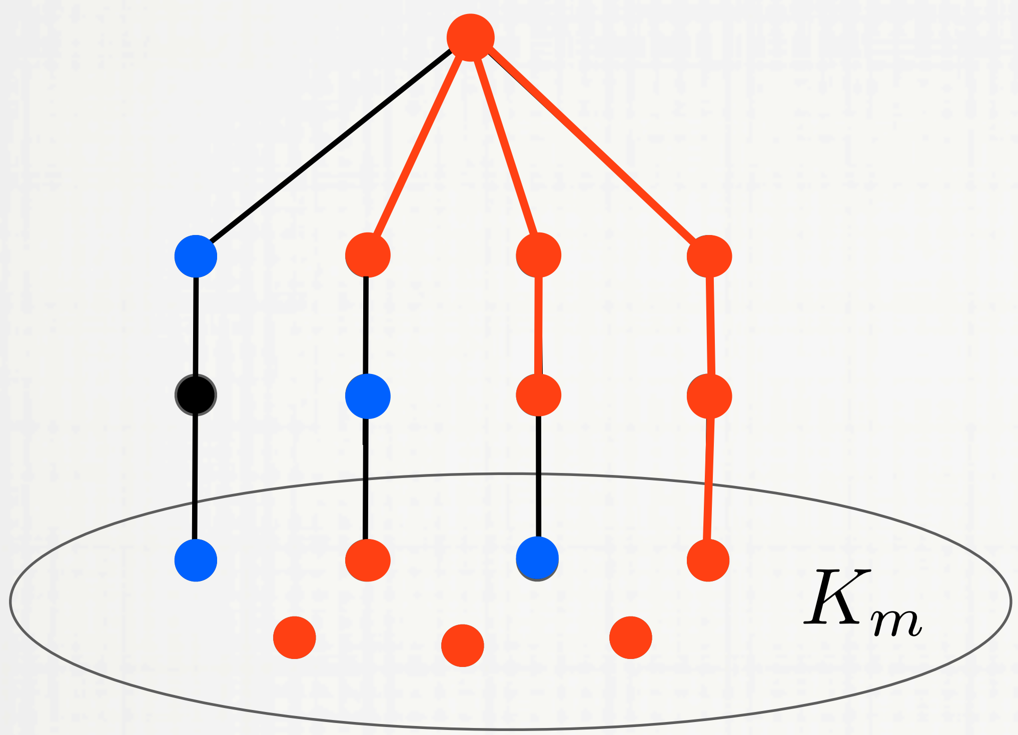

save $\dfrac{5}{m+9}$ vertices

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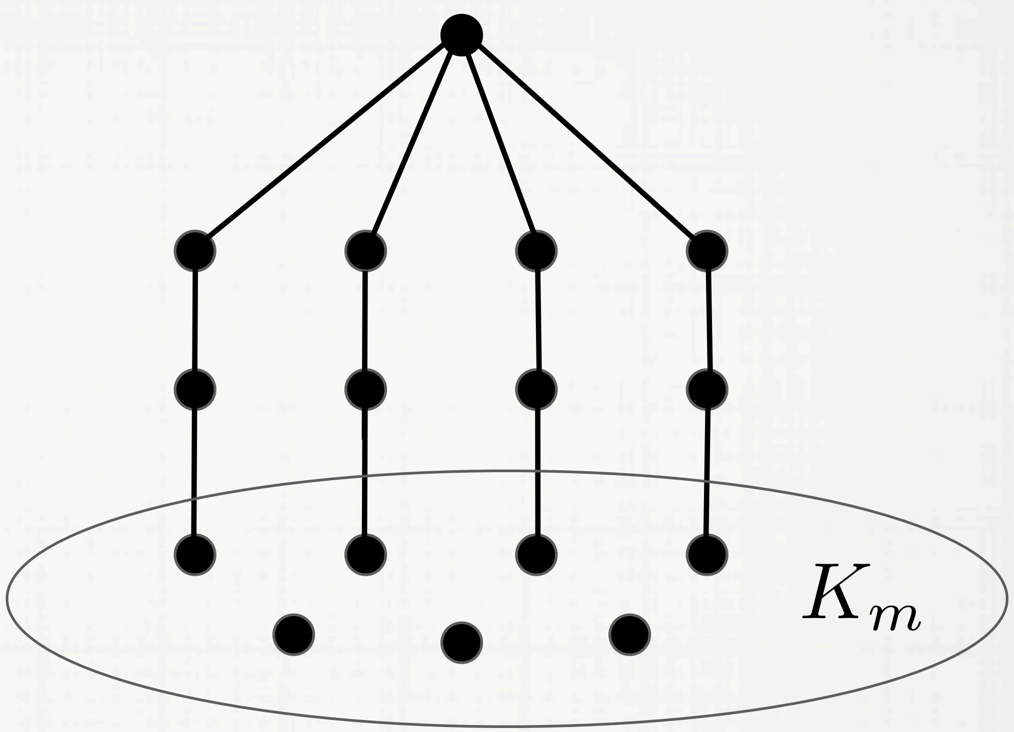

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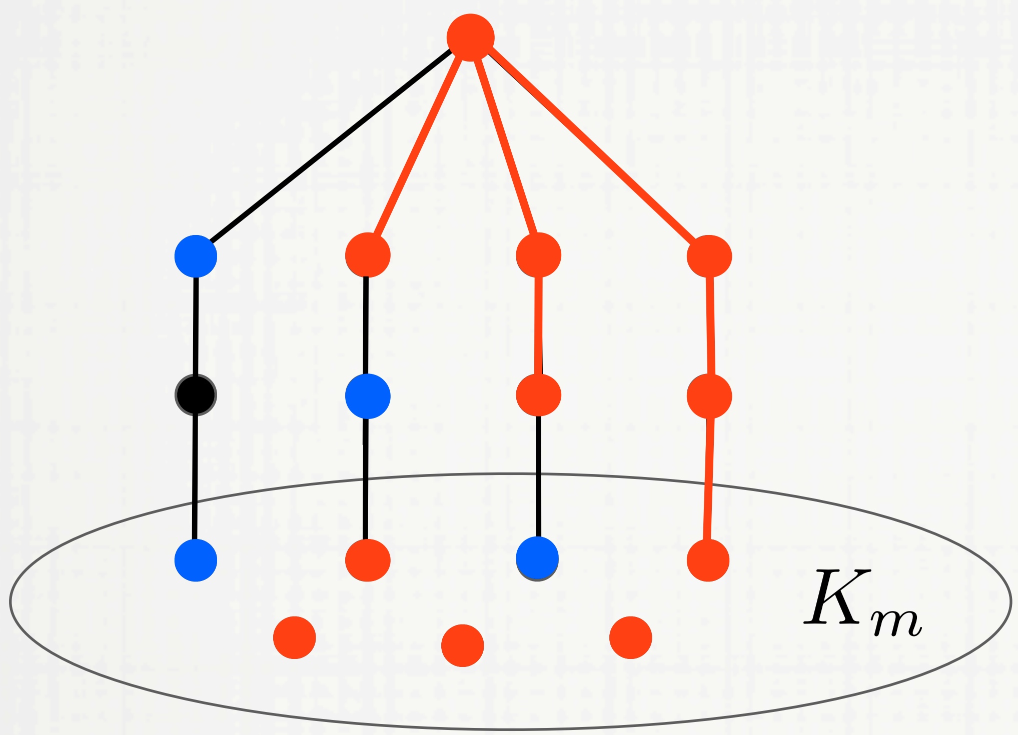

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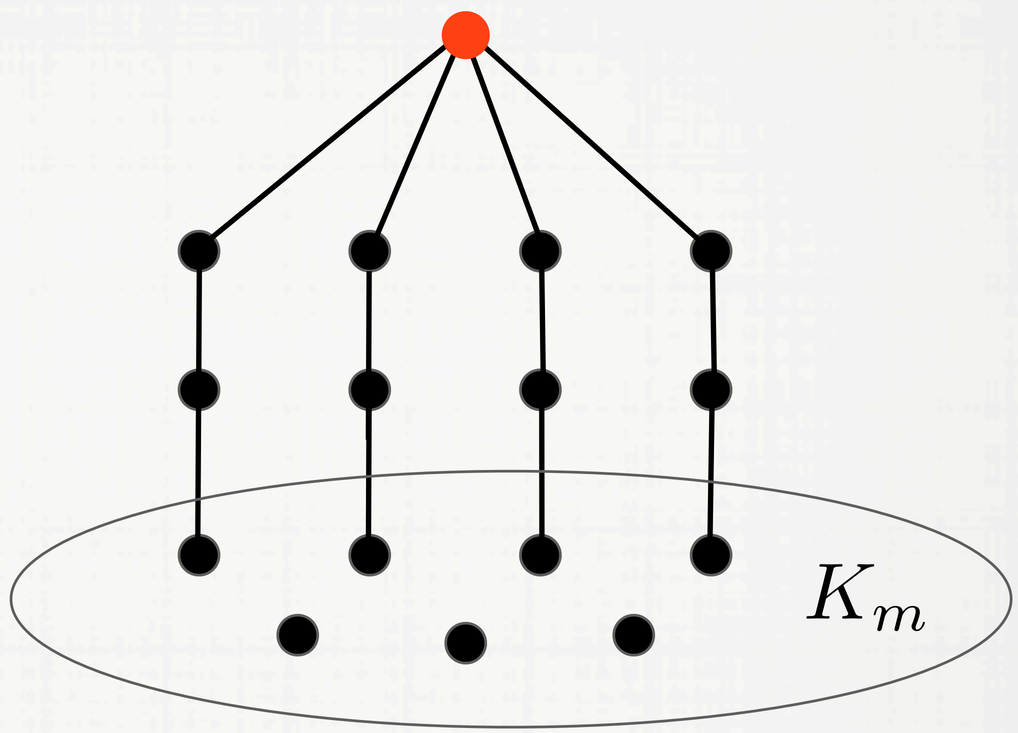

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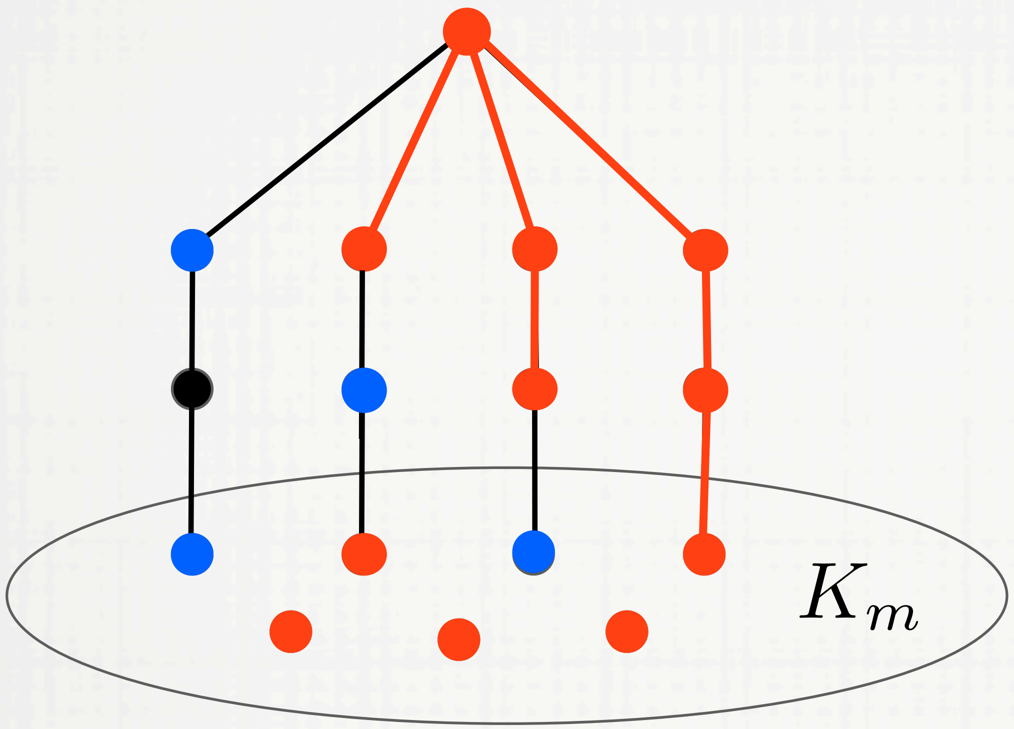

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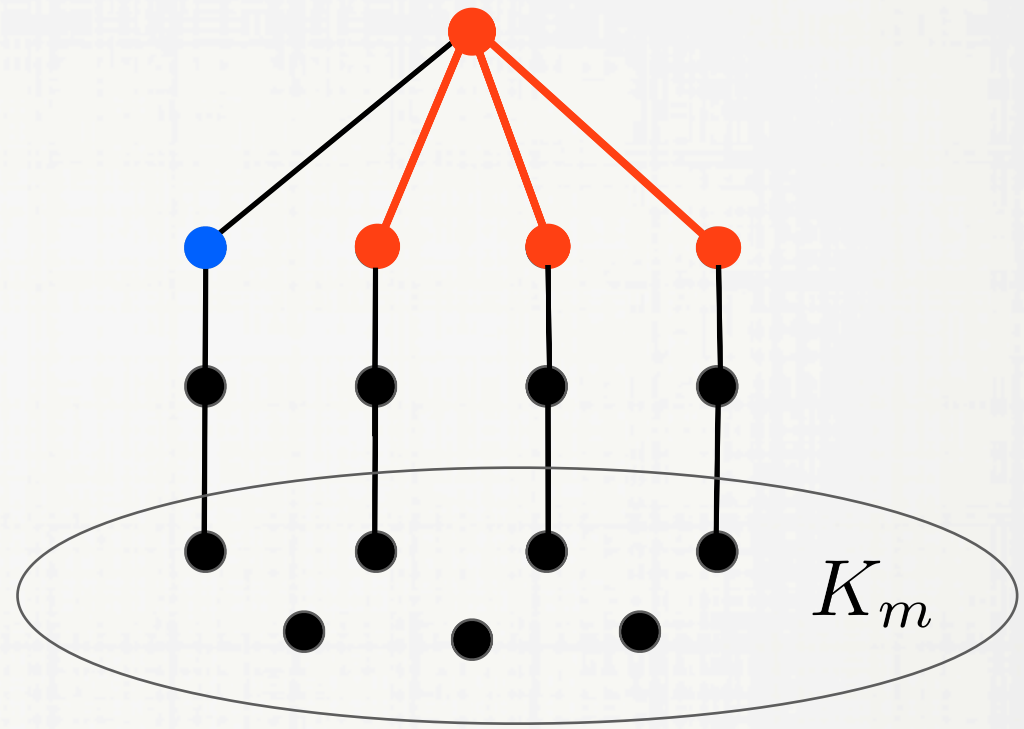

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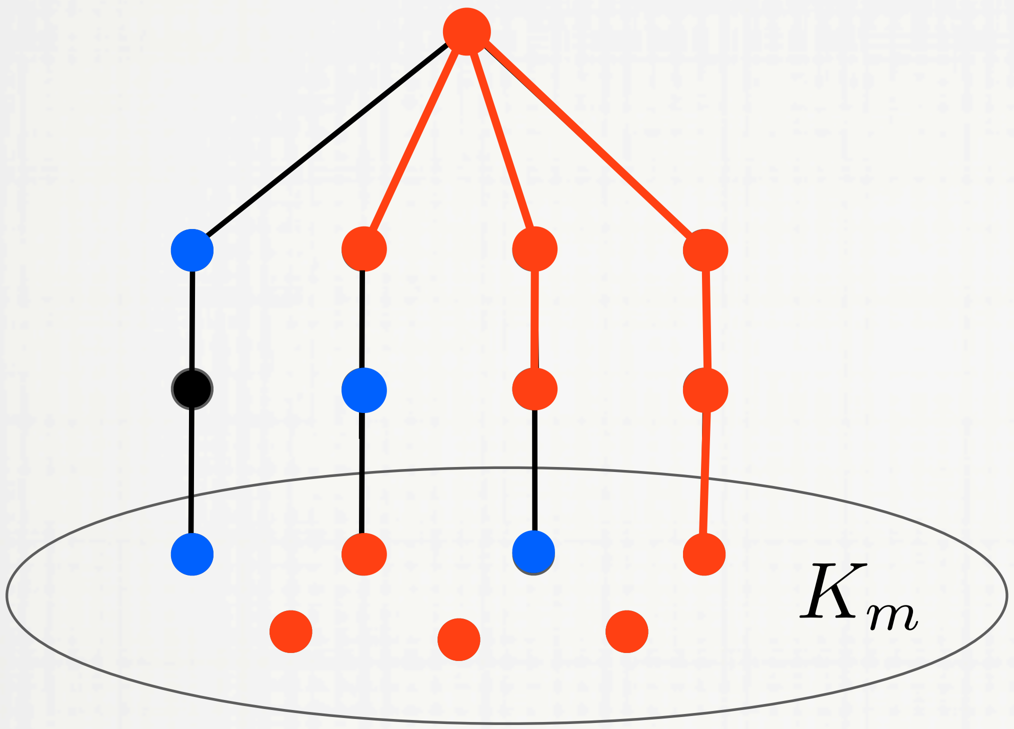

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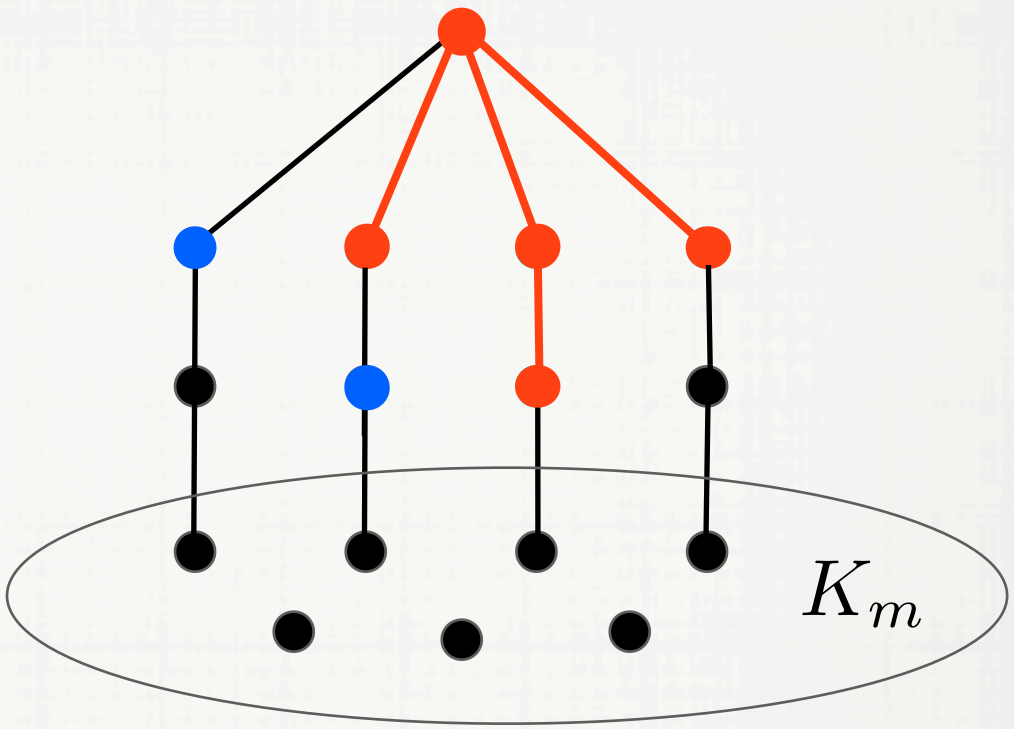

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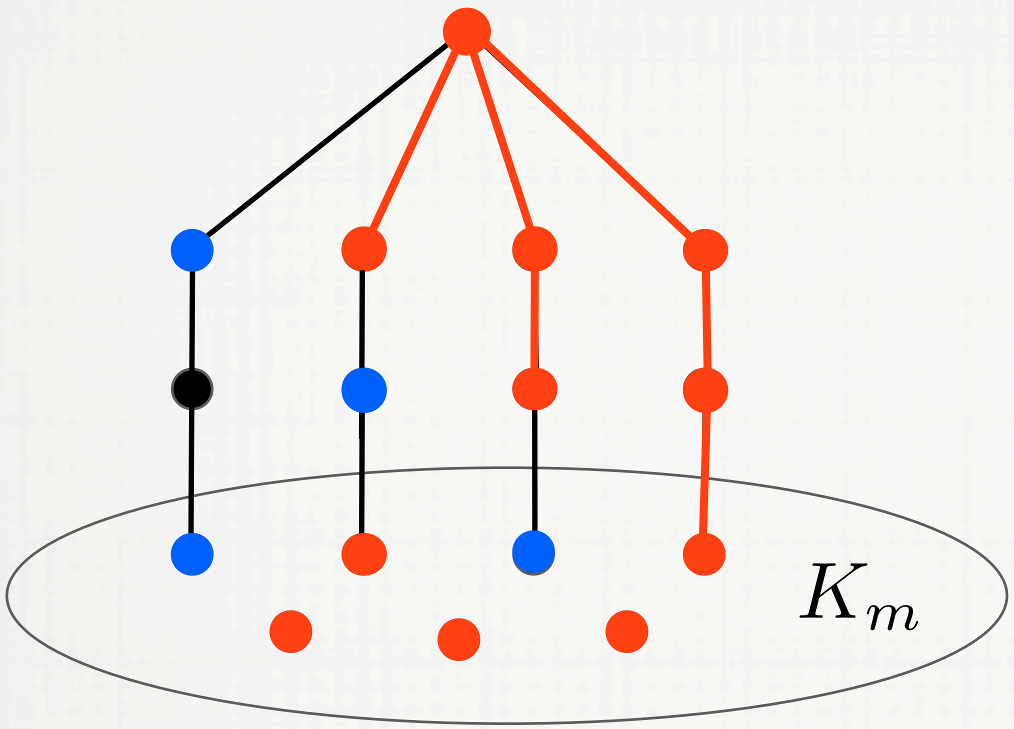

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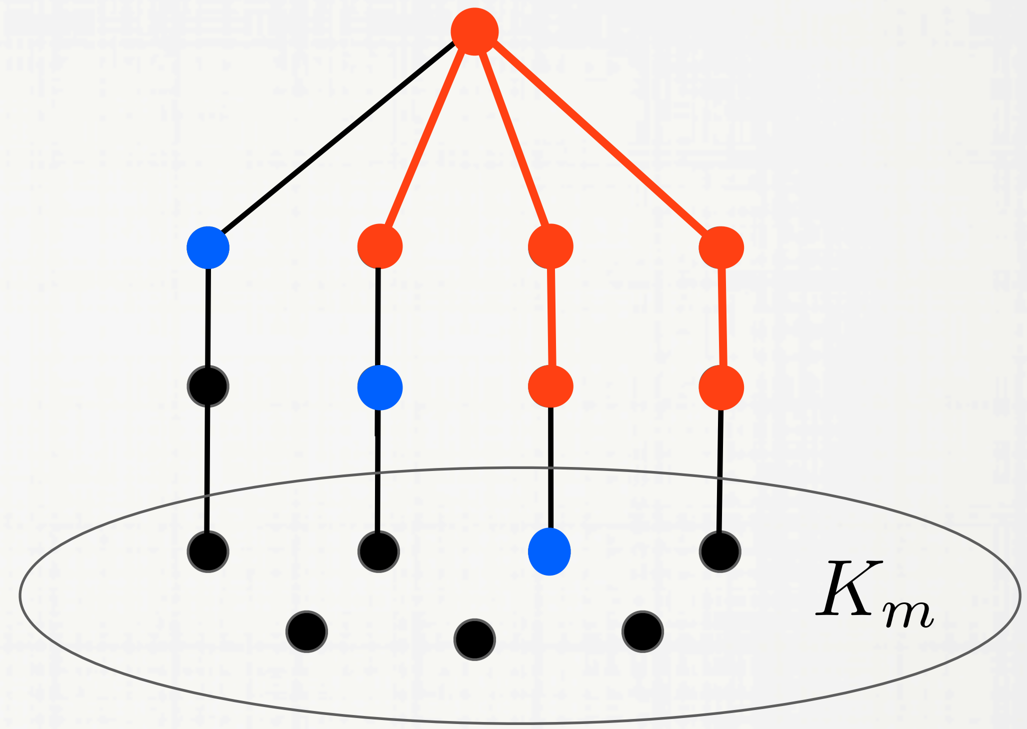

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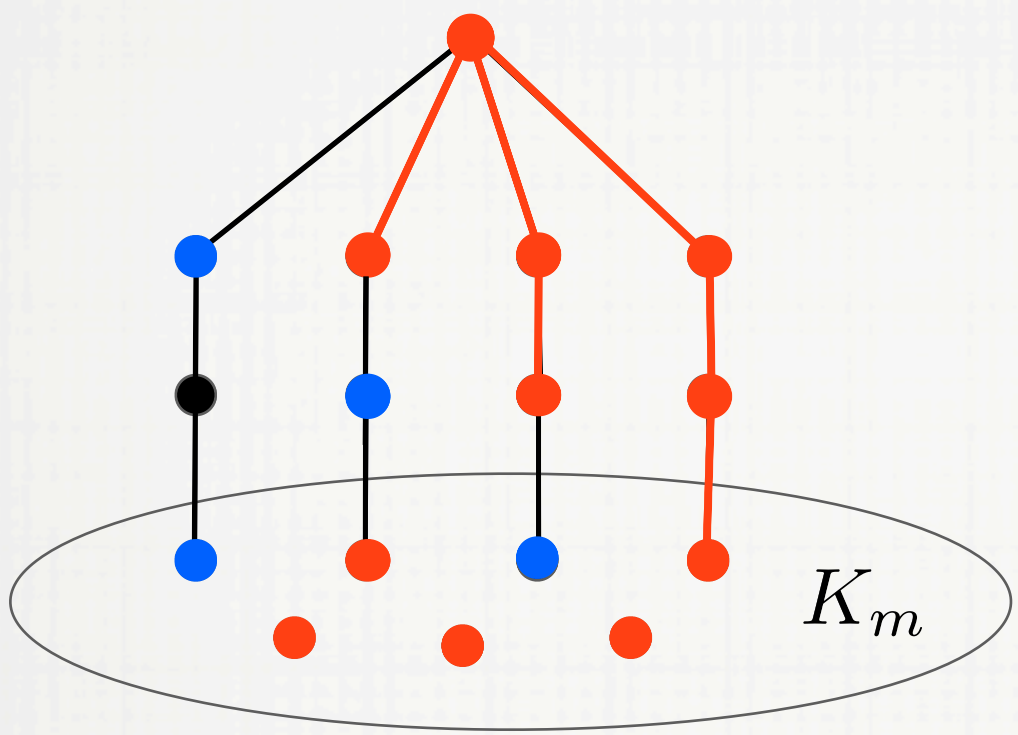
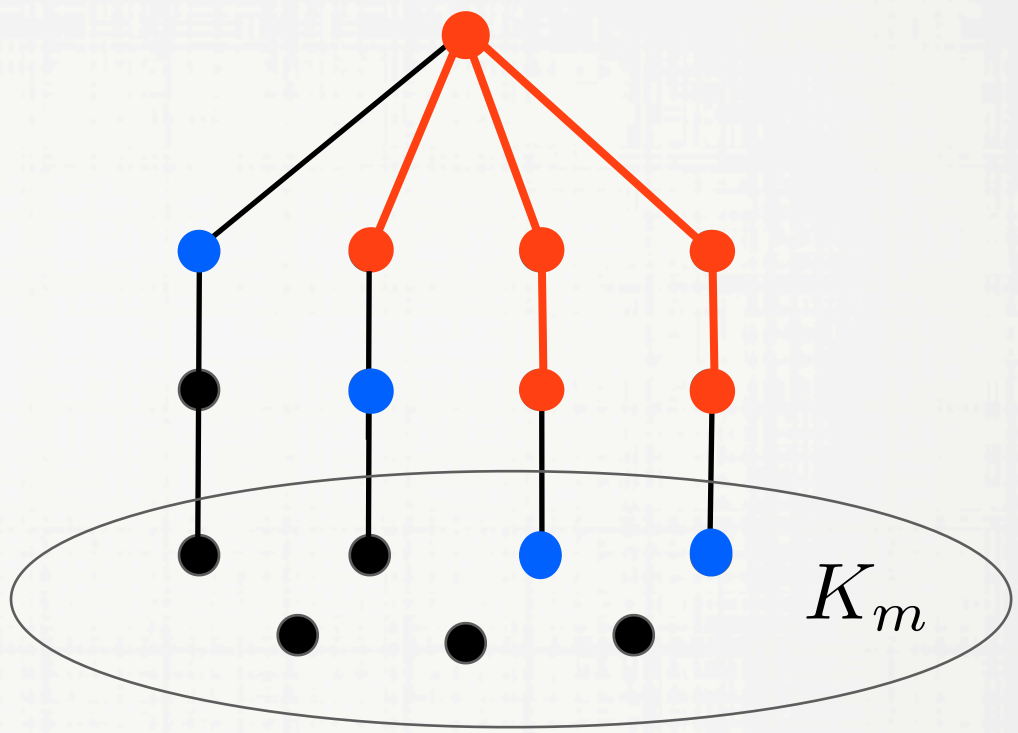


$$\text{save } \frac{5}{m+9} \text{ vertices}$$

The Pyro Game



$$\text{save } \frac{m+3}{m+9} \text{ vertices}$$

Let $MSV_P(G, \nu_0)$ denote the maximum number of vertices of G that can be saved if the pyro initially burns vertex ν_0

<u>Instance</u>: A rooted graph (G, ν_0) and an integer $k \geq 1$.

<u>Question</u>: Is $MSV_P(G, \nu_0) \geq k$?

NP-hard!

For some fixed k, we say the pyro "wins" if he spreads to (burns) a vertex distance k from ν_0 (otherwise the firefighter "wins")

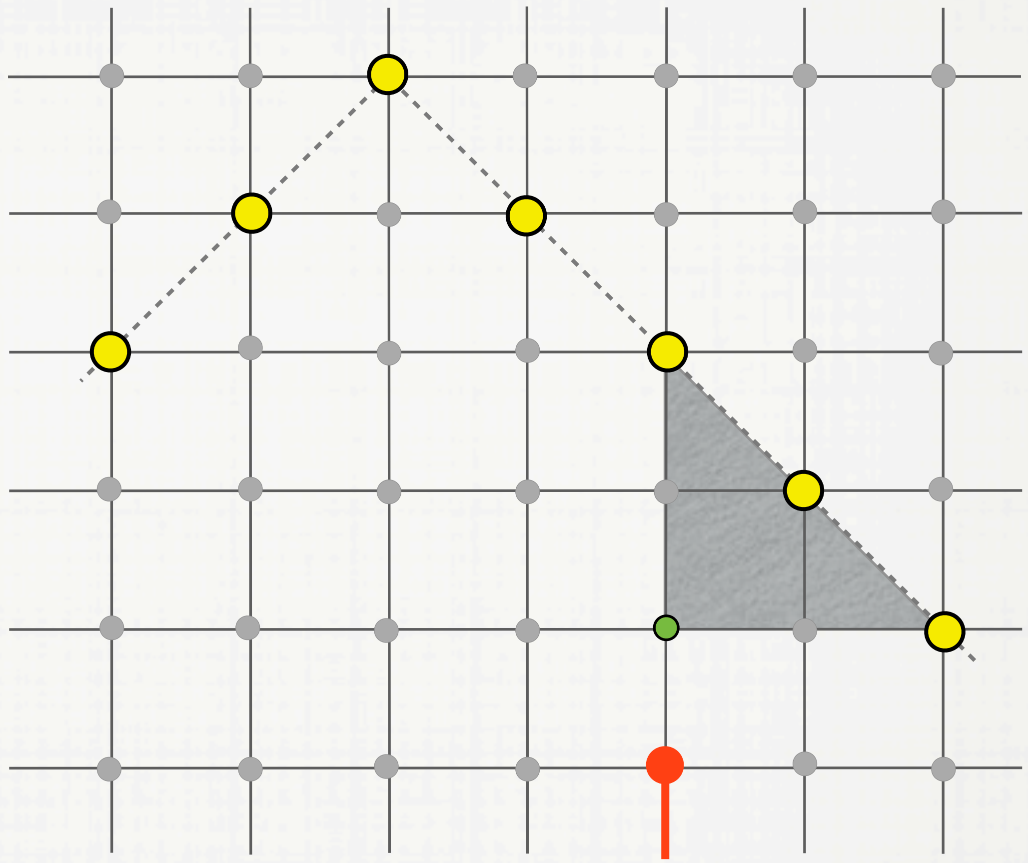
Little Lemma:

If the pyro can win, then there exists a set $\{\nu_0, \nu_1, \ldots, \nu_t\}$ such that for all $1 \leq i \leq t$, ν_i is initially burned during step $i-1$ and the pyro spreads from ν_i during step i.

Let ν_s be the lowest-indexed vertex such that ν_s was not initially burned during step $s-1$.

Suppose instead ν_s was burned during step $j-1 < s-1$.

Cartesian Grids



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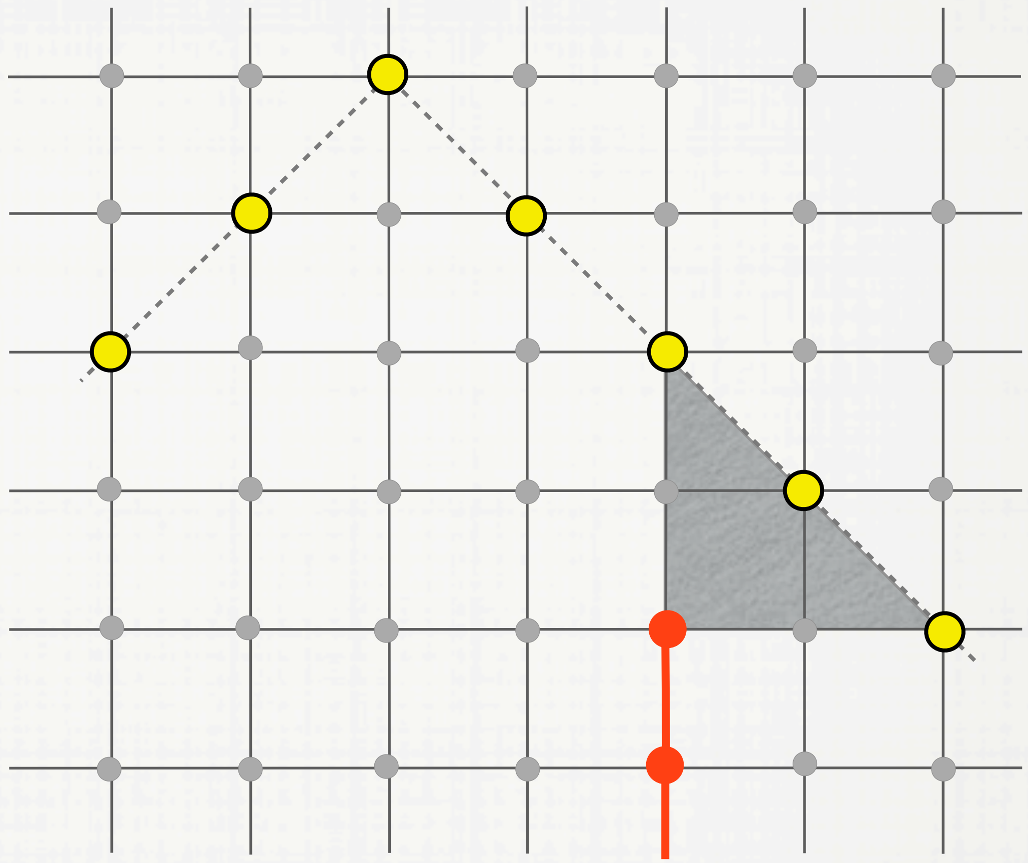

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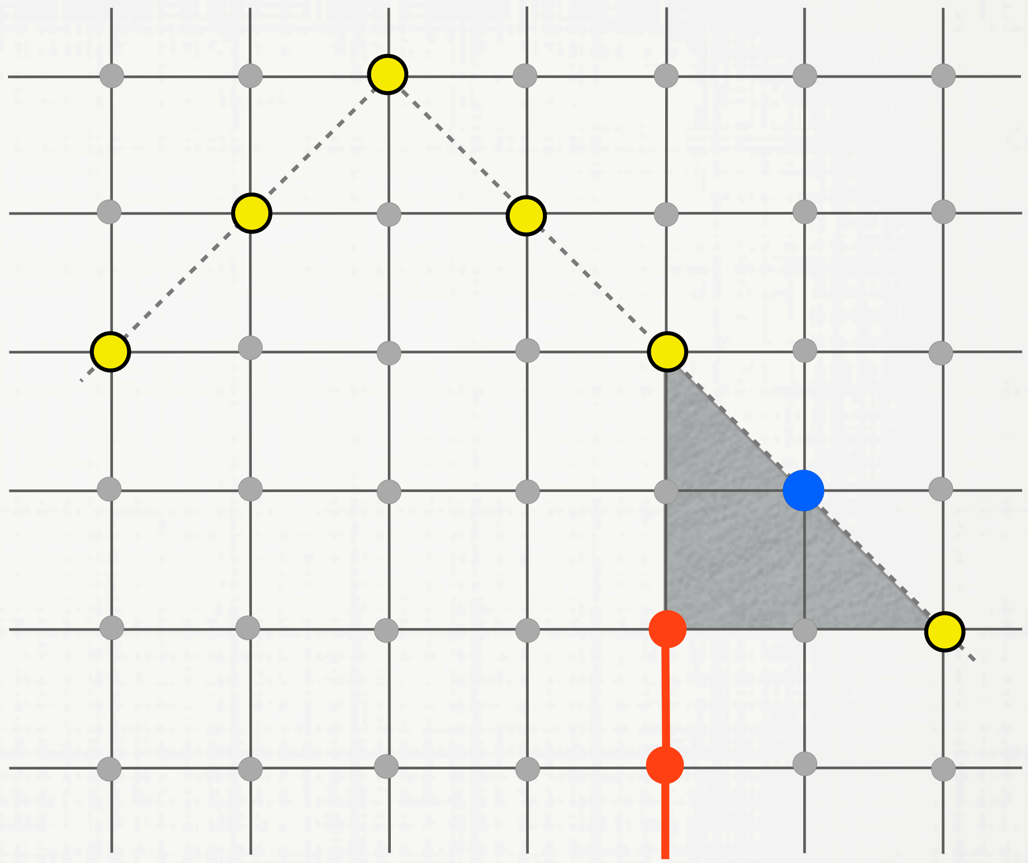

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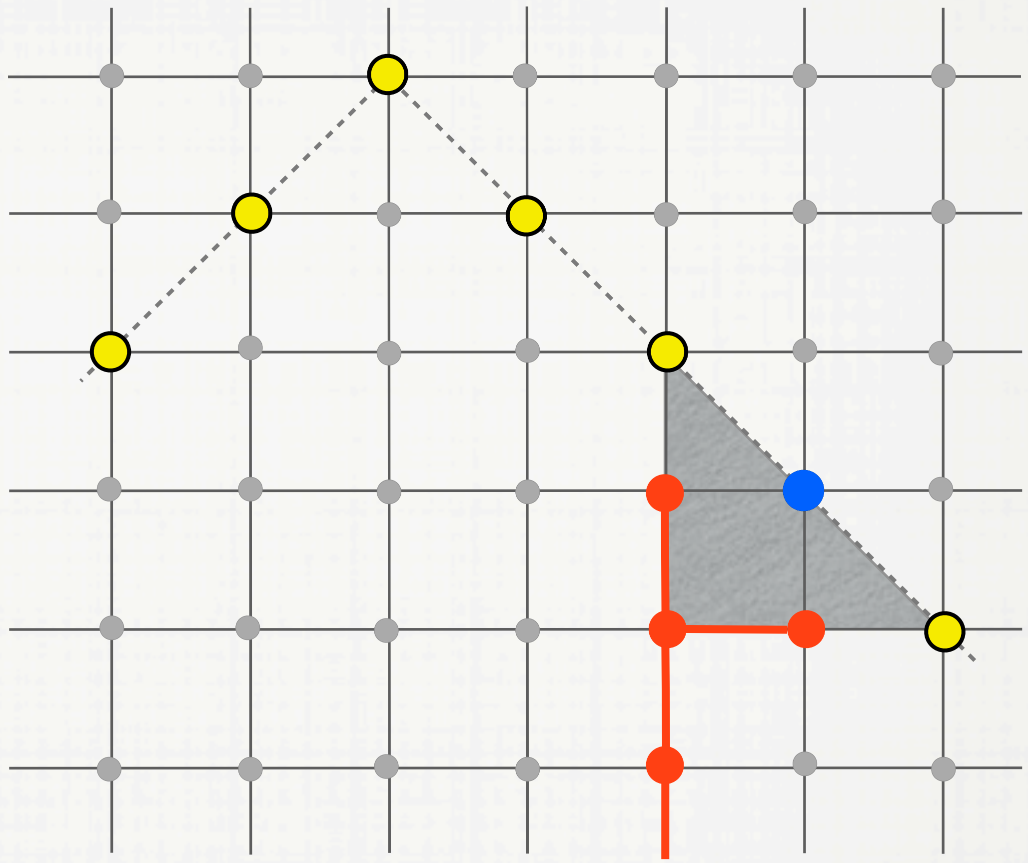

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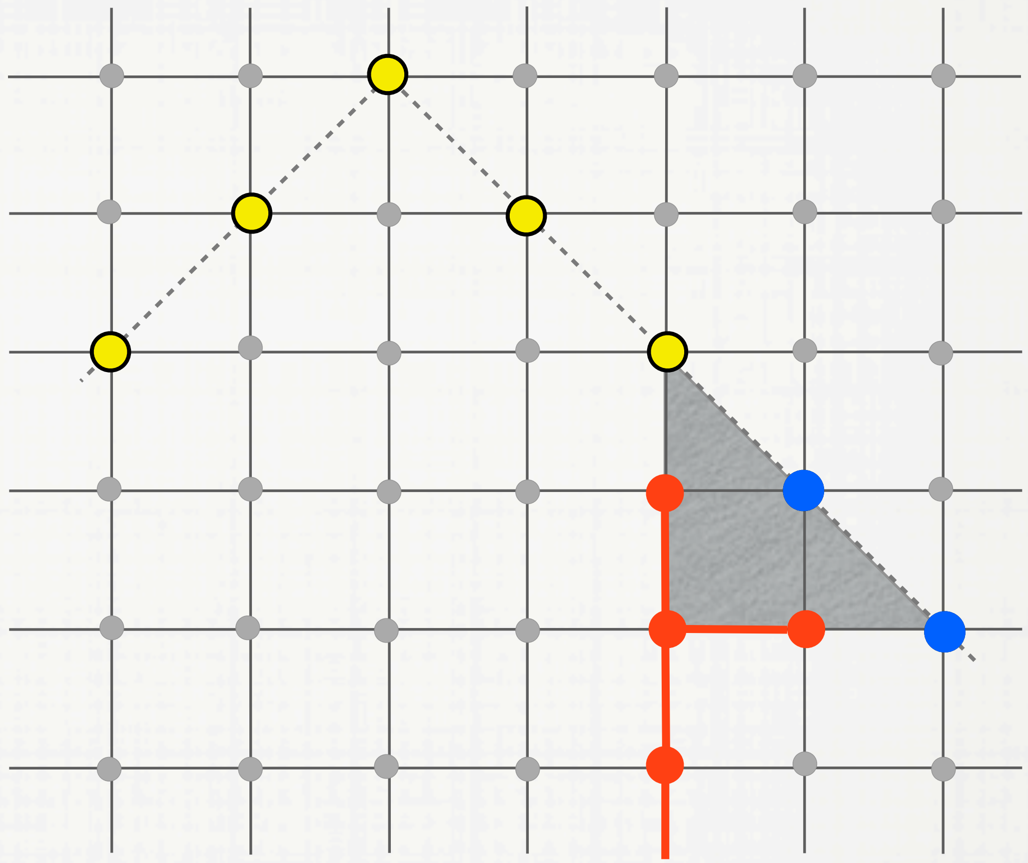

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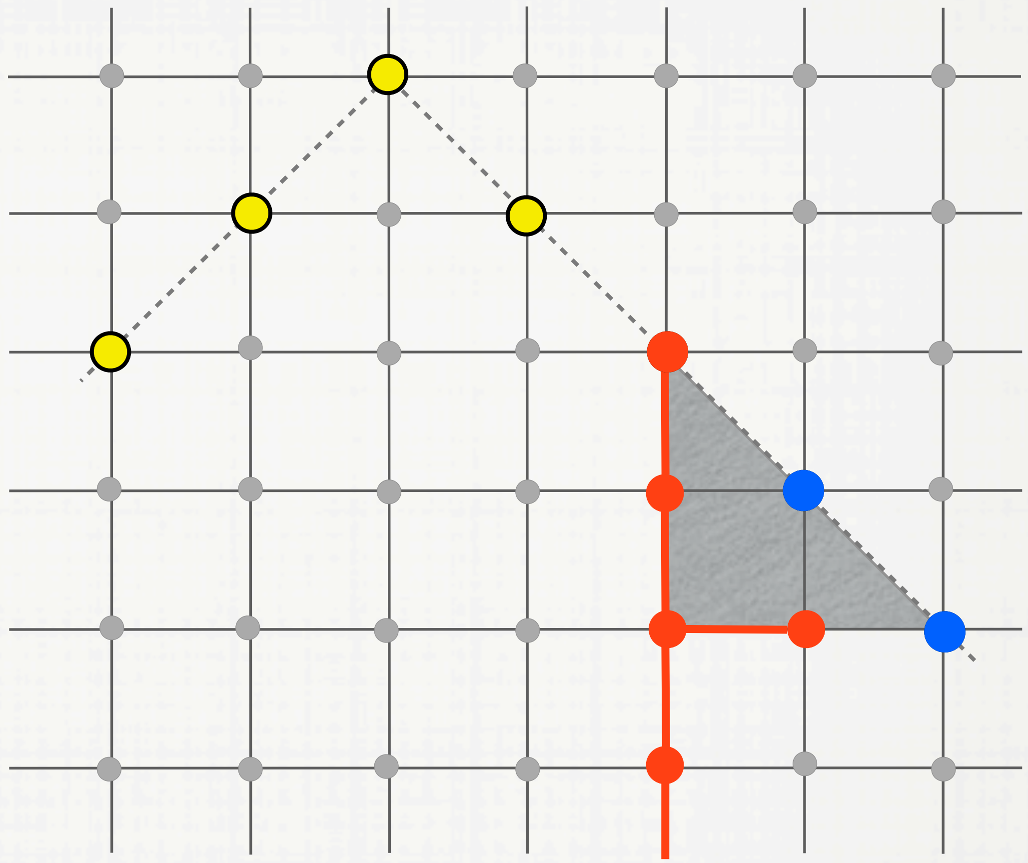
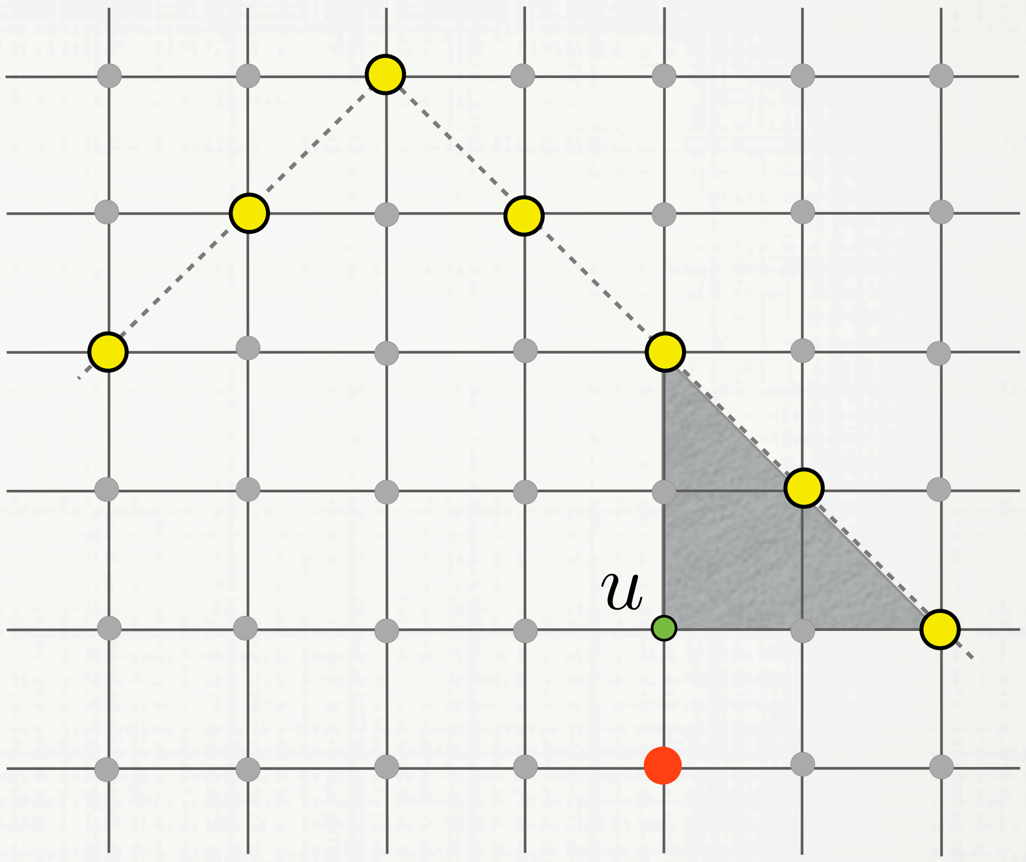


Cartesian Grids

$$Th_t(u) = 1$$



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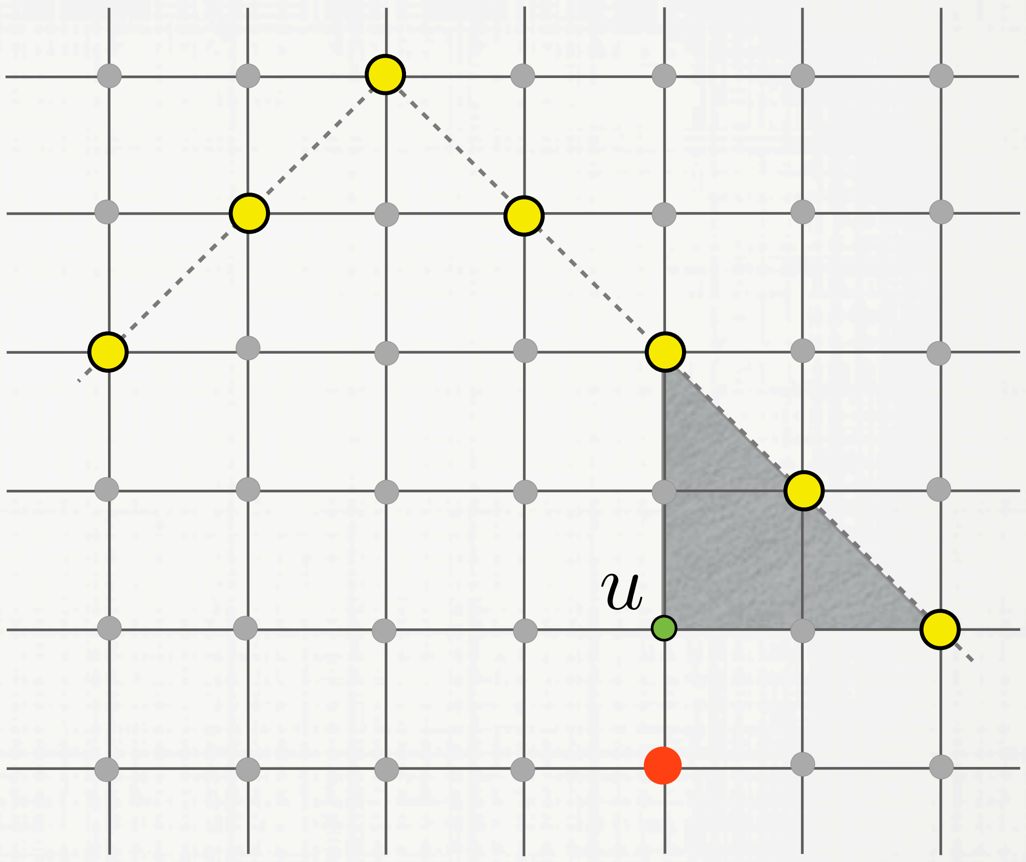

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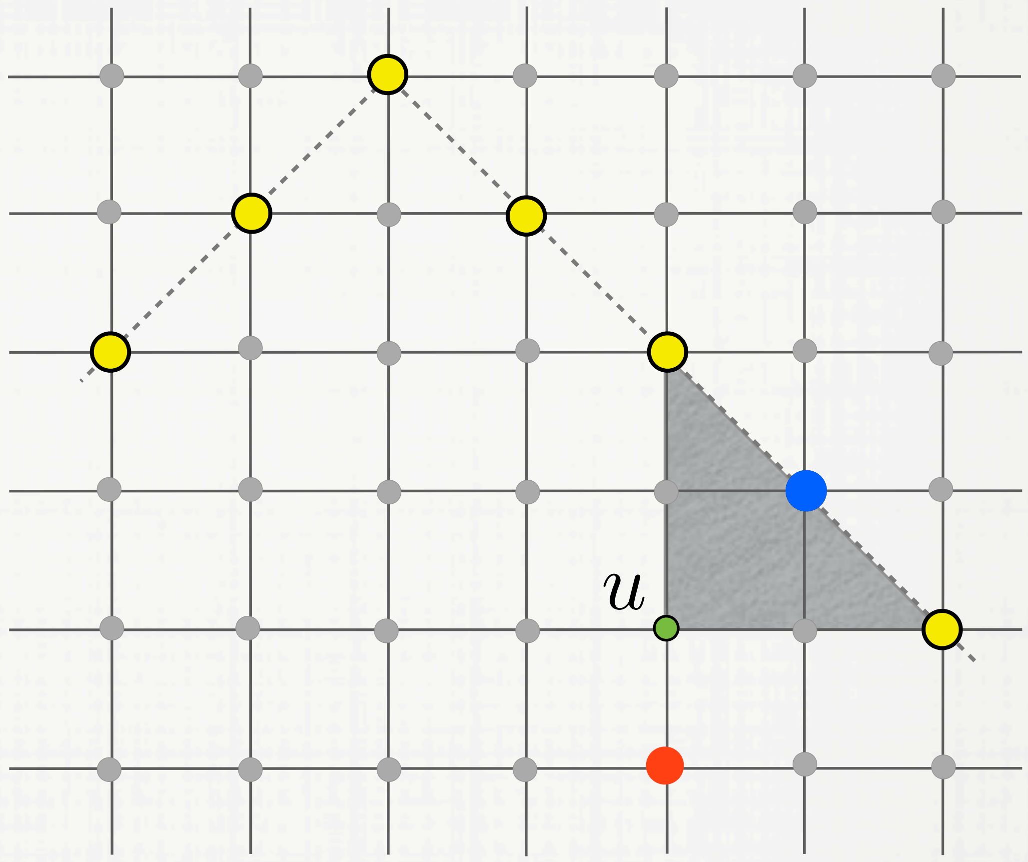
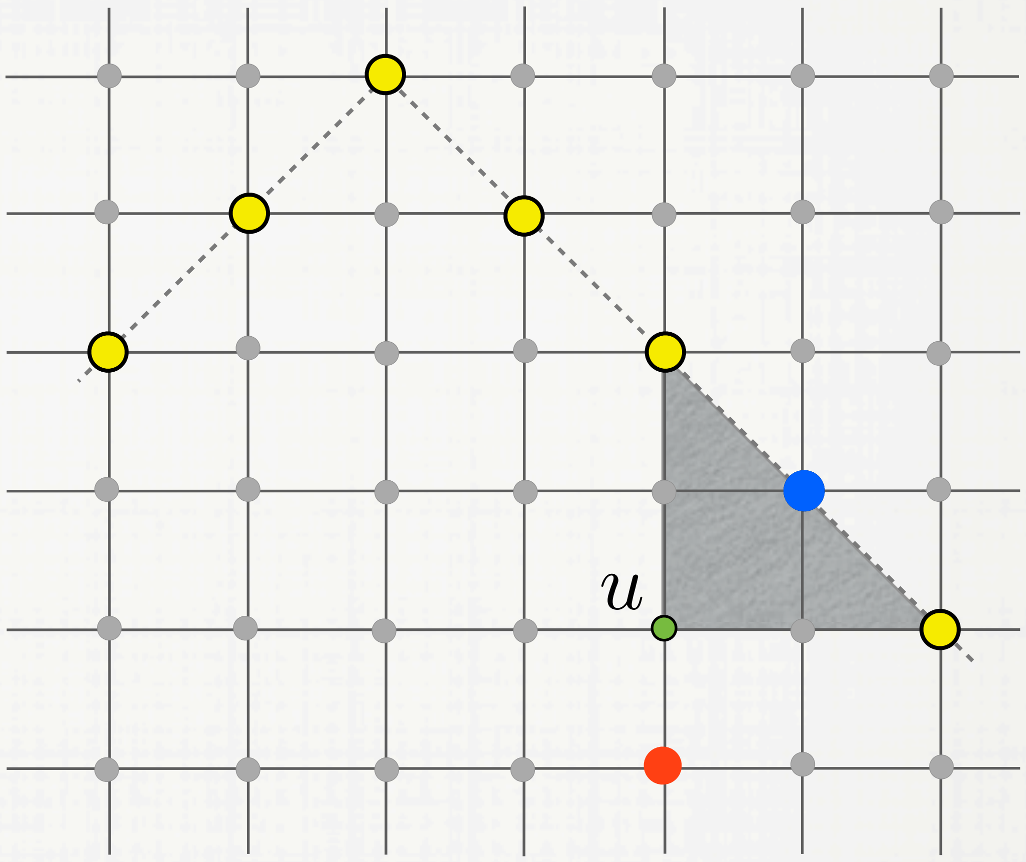


Cartesian Grids

$$\cancel{Th_t(u) = 1}$$

$$Th_t{}'(u) = 0$$



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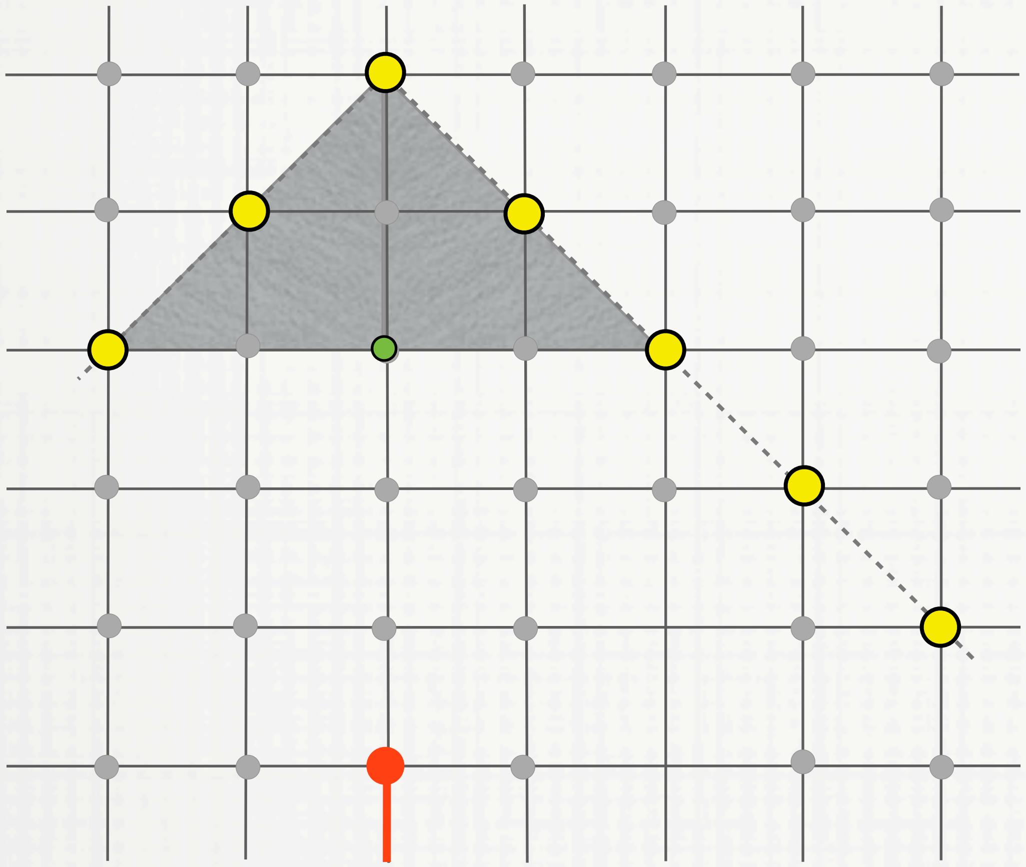

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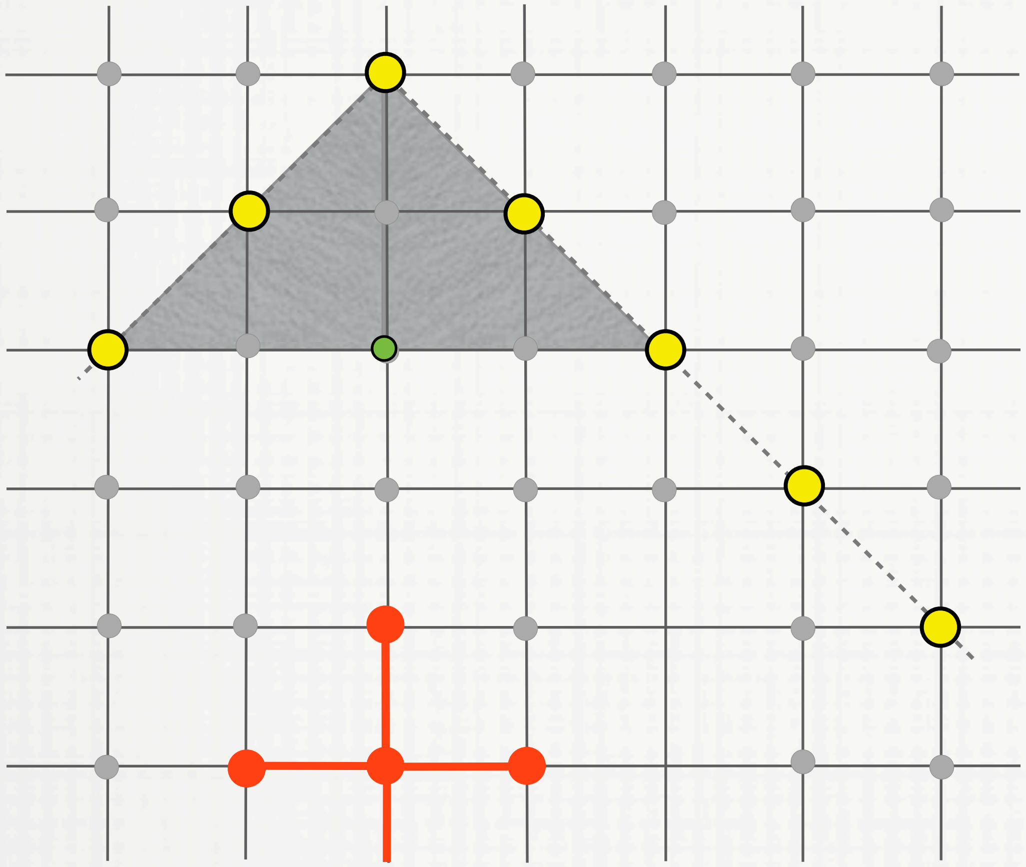

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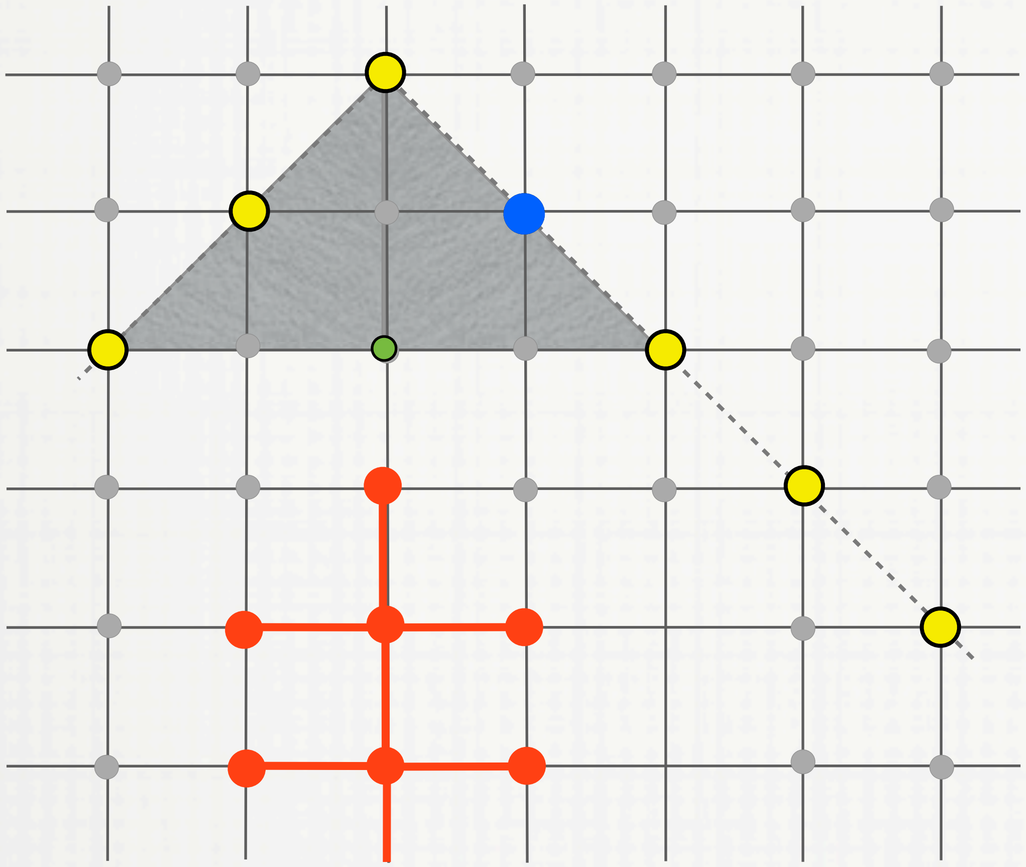

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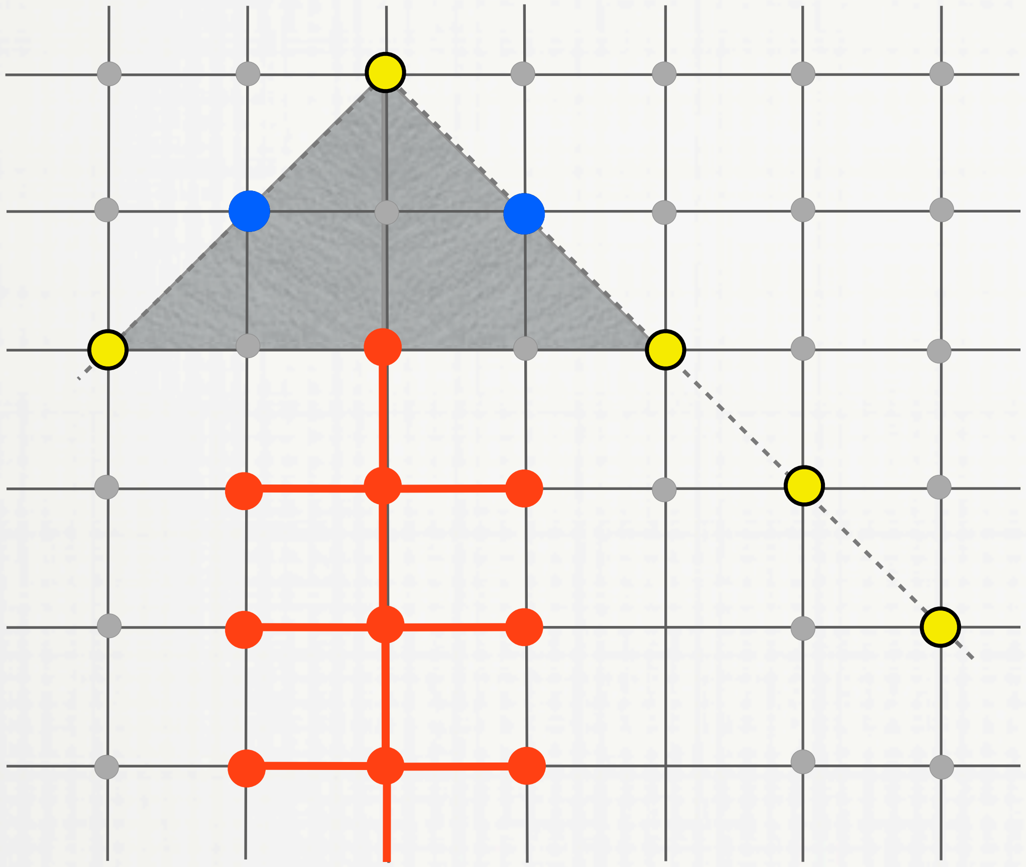

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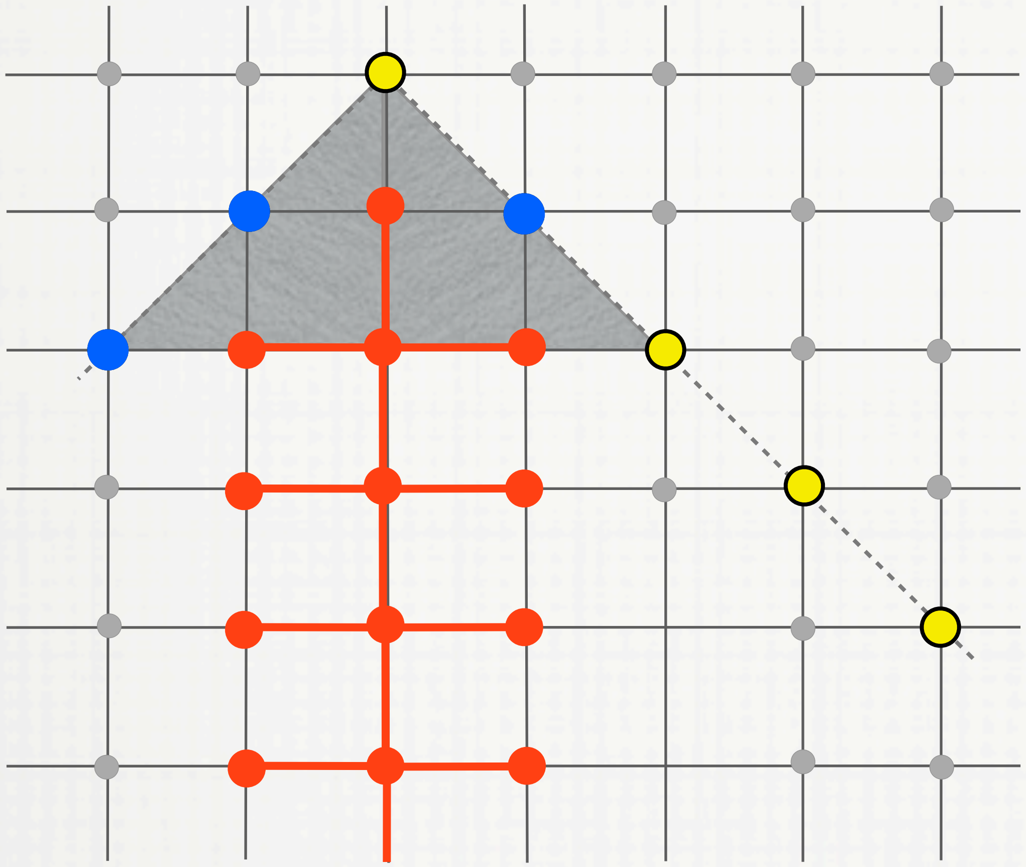

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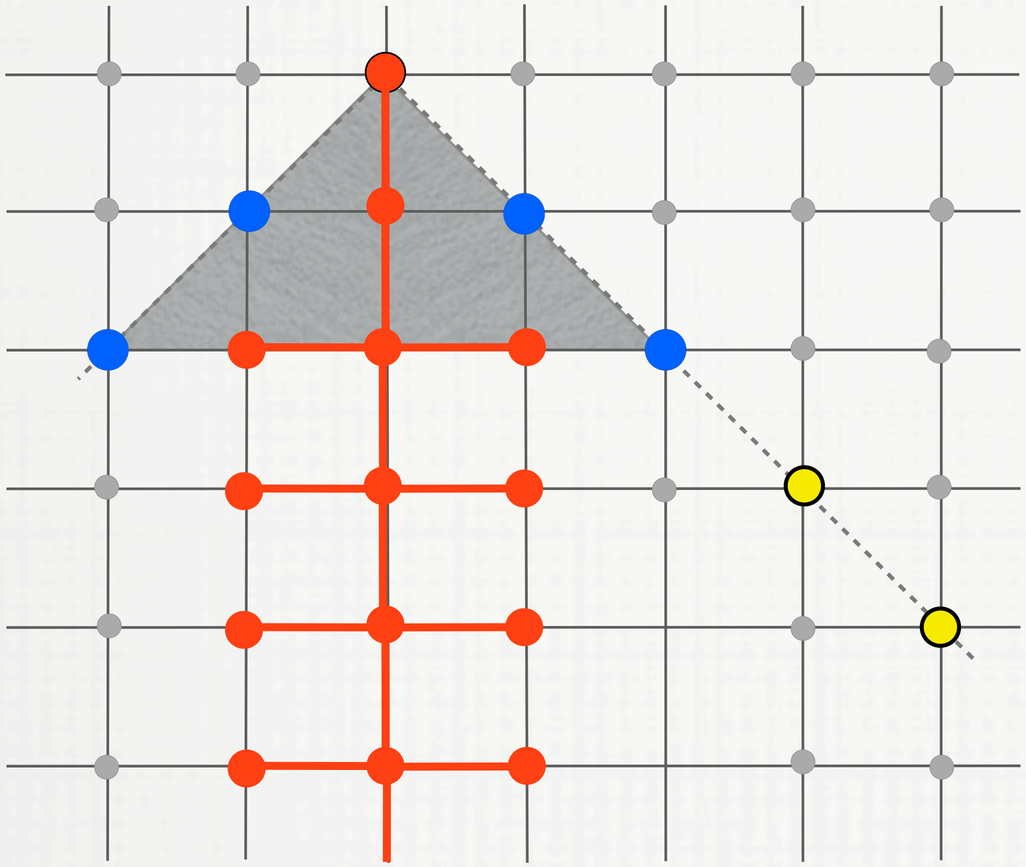

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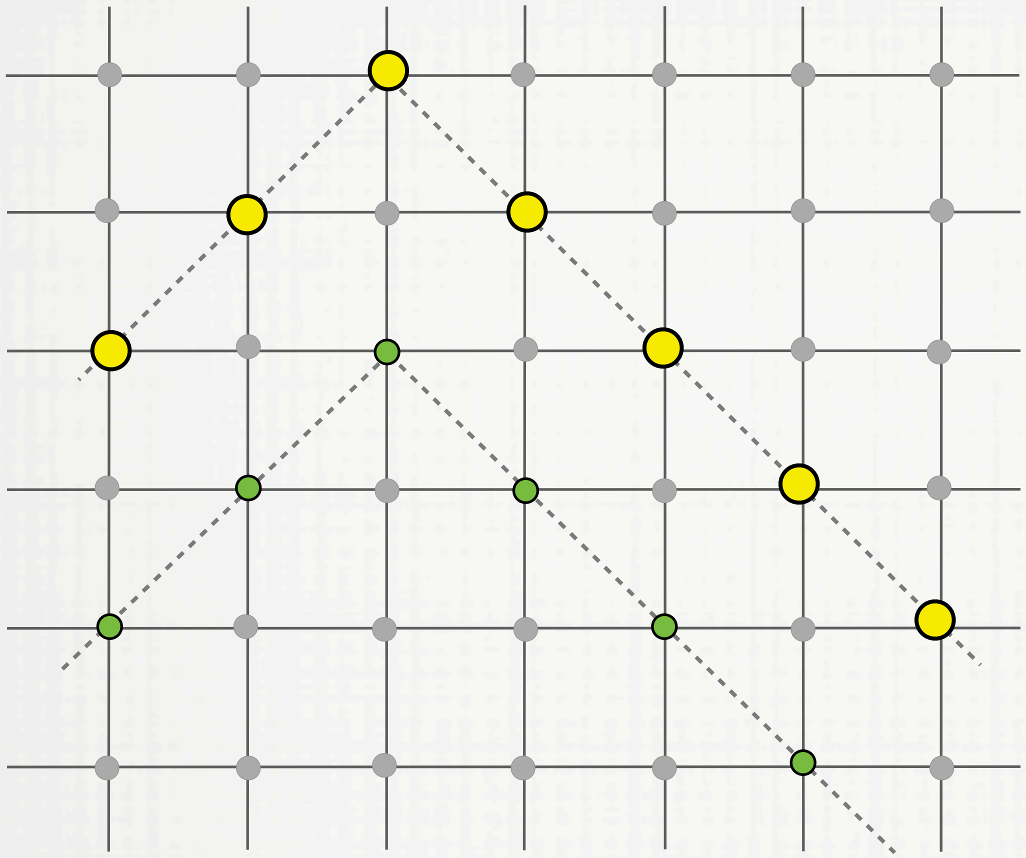

Let $\mathcal{N}_t(u)$ = unprotected boundary vertices in the second neighbourhood at the end of step t

For each $u \in D_5$ we consider

$$\boxed{Th_t(u) = \Big|\mathcal{N}_{t-1}(u)\Big| - \Big(d_{t-1}(u) + 1\Big)}$$

threat value of u at the <u>beginning</u> of step t

$$\boxed{Th_t{}'(u) = \Big|\mathcal{N}_t(u)\Big| - \Big(d_{t-1}(u) + 1\Big)}$$

threat value of u <u>during</u> step t

On the Cartesian Grid, suppose $\nu_0 = (0, 0)$

Theorem:

On the Cartesian Grid, the firefighter can contain the pyro such that all burned vertices are distance 6 or less from the origin.

Present an algorithm to determine the vertex to protect at each step

Prove that for each step $t \geq 3$, $Th_t{}'(u) \leq 0$ for all $u \in D_5$

Suppose otherwise:

there is a step t when $Th_t{}'(u) \geq 1$ for some $u \in D_5$

$Th_t(u) \geq 2$
for some $u \in D_5$

$Th_t(u) \geq 1; Th_t(v) \geq 1$

$\mathcal{N}_{t-1}(u) \cap \mathcal{N}_{t-1}(v) = \emptyset$

for some $u, v \in D_5$

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Thank you!

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