9th Graph Searching in Canada (GRASCan) Workshop

August 4–6, 2021

Online

Wednesday, August 4

Online summer school on graph searching

All start times EST/EDT.

9:00	Jesse Geneson, *Introduction to graphs*
11:00	Eunjeong Yi, *On the metric dimension of graphs*
1:00	Cong Kang, *On graph Laplacian*
3:00	Shen-Fu Tsai, *Graph pursuit games*
5:00	Boris Brimkov, *Optimal monitoring of electrical grids*

Thursday, August 5

All start times EST/EDT.

8:45 - 9:00	Opening remarks
9:00 - 10:00	**Margaret-Ellen Messinger**, Mount Allison University *New variants and open problems in graph searching*
10:00 - 10:30	COFFEE
10:30 - 11:00	**Nicolas Nisse**, INRIA Sophia Antipolis *Minimum lethal sets in grids and tori under 3-neighbour bootstrap percolation*
11:00 - 11:30	**Thanasis Kehagias**, Aristotle University of Thessaloniki *Generalized Cops and Robbers: Multi-Player Pursuit Games on Graphs*
11:30 - 12:00	**Nikolas Townsend**, University of Rhode Island *Catching an Infinite-Speed Robber on Grid Graphs*
12:00 - 1:30	LUNCH BREAK
1:30 - 2:00	**Jérémie Turcotte**, McGill University *New bounds on the cop number of abelian Cayley graphs*
2:00 - 2:30	**Thomas Lidbetter**, Rutgers University *Continuous patrolling games*
2:30 - 3:00	**Trent G. Marbach**, Ryerson University *The localization number of graph classes with small diameter*
3:00 - 3:30	**Brittany Pittman**, Ryerson University *The localization capture time of a graph*
3:30 - 4:30	OPEN DISCUSSION

Friday, August 6

9:00 - 10:00	**Alan Frieze**, Carnegie Mellon University *Maker Breaker on Digraphs*
10:00 - 10:30	COFFEE
10:30 - 11:00	**Fionn Mc Inerney**, CISPA Helmholtz Center for Information Security *The Largest Connected Subgraph Game*
11:00 - 11:30	**Daniela Ferrero**, Texas State University *Product Throttling Power Domination*
11:30 - 12:00	**Brendan Sullivan**, Emmanuel College *Two Graph Products (Cartesian and Strong) and the Cops and Robber Game*
12:00 - 1:30	LUNCH BREAK
1:30 - 2:00	**Shahin Kamali**, University of Manitoba *Algorithms for Burning Graph families*
2:00 - 2:30	**Seyyed Aliasghar Hosseini**, Simon Fraser University *W-Active Cops and Robber*
2:30 - 3:30	OPEN DISCUSSION

Abstracts of Talks

Margaret-Ellen Messinger, Mount Allison University — Thursday
New variants and open problems in graph searching — 9:00 - 10:00

This talk will present results from several graph searching problems, including Cops and Robber, Graph Cleaning, and Firefighting. In the spirit of encouraging collaborations, which is a main goal of GRASCan workshops, a large portion of the talk will focus on presenting a number of new open problems. My aim is to convince the audience that these problems are both interesting and eminently solvable!

Nicolas Nisse, INRIA Sophia Antipolis — Thursday
Minimum lethal sets in grids and tori under 3-neighbour bootstrap percolation — 10:30 - 11:00

Let $r \geq 1$ be any non negative integer and let $G = (V, E)$ be any undirected graph in which a subset $D \subseteq V$ of vertices are initially *infected*. We consider the process in which, at every step, each non-infected vertex with at least r infected neighbours becomes infected and an infected vertex never becomes non-infected. The problem consists in determining the minimum size $s_r(G)$ of an initially infected vertices set D that eventually infects the whole graph G. This problem is closely related to cellular automata, to percolation problems and to the Game of Life studied by John Conway. Note that $s_1(G) = 1$ for any connected graph G. The case when G is the $n \times n$ grid, $G_{n\times n}$, and $r = 2$ is well known and appears in many puzzle books, in particular due to the elegant proof that shows that $s_2(G_{n\times n}) = n$ for all $n \in \mathbb{N}$. We study the cases of square grids, $G_{n\times n}$, and tori, $T_{n\times n}$, when $r \in \{3, 4\}$. We show that $s_3(G_{n\times n}) = \lceil \frac{n^2+2n+4}{3} \rceil$ for every n even and that $\lceil \frac{n^2+2n}{3} \rceil \leq s_3(G_{n\times n}) \leq \lceil \frac{n^2+2n}{3} \rceil + 1$ for any n odd. When n is odd, we show that both bounds are reached, namely $s_3(G_{n\times n}) = \lceil \frac{n^2+2n}{3} \rceil$ if $n \equiv 5 \pmod 6$ or $n = 2^p - 1$ for any $p \in \mathbb{N}^*$, and $s_3(G_{n\times n}) = \lceil \frac{n^2+2n}{3} \rceil + 1$ if $n \in \{9, 13\}$. Finally, for all $n \in \mathbb{N}$, we give the exact expression of $s_4(G_{n\times n})$ and of $s_r(T_{n\times n})$ when $r \in \{3, 4\}$. Joint work with Fabricio Benevides, Jean-Claude Bermond and Hicham Lesfari.

Thanasis Kehagias, Aristotle University of Thessaloniki — Thursday
Generalized Cops and Robbers: Multi-Player Pursuit Games on Graphs — 11:00 - 11:30

We introduce and study generalized Cops and Robbers (GCR) *multi-player* games, i.e., N-player pursuit games played in graphs. These are *stochastic discounted N-player pursuit games* with payoff functions which satisfy the *pursuit/evasion semantics*. Special cases of these games include: (a) the classic *Cops and Robbers* (CR) game, (b) Bonato and MacGilivray's generalized *two-player* CR games, (c) the *selfish cops and adversarial robber* (SCAR) game and (d) the *cyclic pursuit game* in which a player is simultaneously a pursuer and evader. We prove that, under mild conditions on the payoff function, each GCR game possesses at least two *Nash Equilibria*: one in positional deterministic strategies and another in non-positional ones. We also relate, for several special cases, the capturing properties of GCR Nash Equilibria to the (classic) *cop-number* of a graph.

Nikolas Townsend, University of Rhode Island — Thursday 11:30 - 12:00
Catching an Infinite-Speed Robber on Grid Graphs

Recently, there has been considerable interest on variants of Cops and Robbers in which the robber is more mobile than the cops. We focus on the *infinite-speed robber* variant of the game, wherein the robber may traverse an arbitrarily long cop-free path on their turn. In this talk, we determine the infinite-speed cop number on two-dimensional Cartesian grid-like graphs up to a small additive constant, and we give asymptotic bounds for several families of higher-dimensional Cartesian grids. This is joint work with William B. Kinnersley.

Jérémie Turcotte, McGill University — Thursday 1:30 - 2:00
New bounds on the cop number of abelian Cayley graphs

We consider the game of Cops and robbers on abelian Cayley graphs. By refining the techniques of Frankl, Hamidoune and Bradshaw, we show that the cop number of any undirected abelian Cayley graph is at most $\frac{1}{\sqrt{(\sqrt{2}-1)e}}\sqrt{n}+\frac{7}{2}\approx 0.9424\sqrt{n}+\frac{7}{2}$, where n is the number of vertices of the graph. A similar argument shows an upper bound of $\sqrt{\frac{2}{(\sqrt{2}-1)e}}\sqrt{n}+2\approx 1.3328\sqrt{n}+2$ for directed abelian Cayley graphs. We also construct families of undirected and directed abelian Cayley graphs with cop number respectively $\left\lceil\frac{1}{2}\sqrt{n}\right\rceil$ and $\sqrt{n}$. This shows that our upper bounds are tight up to a relatively small multiplicative factor, and yields new Meyniel extremal families. Joint work with Peter Bradshaw and Seyyed Aliasghar Hosseini.

Thomas Lidbetter, Rutgers University — Thursday 2:00 - 2:30
Continuous patrolling games

The game studied here was first proposed in a 2011 paper by Alpern, Morton and Papadaki, which studied a discrete time game where facilities to be protected were modeled as the nodes of a graph. Here we consider protecting roads or pipelines, modeled as the arcs of a continuous network Q. The Attacker chooses a point of Q to attack during a chosen time interval of fixed duration (the attack time, a). The Patroller chooses a unit speed path on Q and intercepts the attack (and wins) if she visits the attacked point during the attack time interval. Solutions to the game have previously been given in certain special cases. Here, we analyze the game on arbitrary networks. Our results include the following: (i) a solution to the game for any network Q, as long as a is sufficiently short, generalizing the known solutions for circle or Eulerian networks; (ii) a solution to the game for all tree networks that satisfy a condition on their extremities. We present a conjecture on the solution of the game for arbitrary trees and establish it in certain cases. This is joint work with Steve Alpern, Katerina Papadaki and Christy Bui.

Trent G. Marbach, Ryerson University — Thursday 2:30 - 3:00
The localization number of graph classes with small diameter

In the localization game played on a graph, a number of cops are tasked with determining the location of an invisible robber via distance probes. The localization number of a graph G, written $\zeta(G)$, is the minimum number of cops needed to ensure the robber's capture. We will present recent work which finds the localization number for a variety of families of graphs with small diameter. One such class are the incidence graphs of certain designs, which are an important class of graphs in pursuit-evasion games as they are Meyniel extremal. We explore other classes such as Kneser graphs and Moore graphs, and their connection to perfect matchings of hypergraphs. This is joint work with Anthony Bonato, and Melissa A. Huggan.

Brittany Pittman, Ryerson University — Thursday 3:00 - 3:30
The localization capture time of a graph

The localization game is a graph searching game analogous to Cops and Robbers, but where the robber is invisible, and the cops send distance probes in an attempt to identify the location of the robber. We present a graph parameter called the capture time, which measures the number of rounds in an instance of the localization game assuming optimal play. We consider graphs for which the capture time is linear in the order of the graph, and show that this holds for trees and interval graphs. We study monotonicity properties on induced subgraphs of capture time, and give bounds for the parameter on trees and multipartite graphs. New bounds on the localization number and capture time are given using treewidth. We finish with an investigation of capture time on the incidence graphs on designs, and bounds are given in the case of projective planes.

Alan Frieze, Carnegie Mellon University — Friday 9:00 - 10:00
Maker Breaker on Digraphs

We study two biased Maker-Breaker games played on the complete digraph $\vec{K}_n$. In the strong connectivity game, Maker wants to build a strongly connected subgraph. We determine the asymptotic optimal bias for this game viz. $\frac{n}{\log n}$. In the Hamiltonian game, Maker wants to build a Hamiltonian subgraph. We determine the asymptotic optimal bias for this game up to a constant factor. Joint with Wesley Pegden.

Fionn Mc Inerney, CISPA Helmholtz Center for Information Security — Friday 10:30 - 11:00
The Largest Connected Subgraph Game

In this talk, the *largest connected subgraph game* played on an undirected graph G is introduced. In each round, Alice first colours an uncoloured vertex of G red, and then, Bob colours an uncoloured vertex of G blue, with all vertices initially uncoloured. Once all the vertices are coloured, Alice (Bob, resp.) wins if there is a red (blue, resp.) connected subgraph whose order is greater than the order of any blue (red, resp.) connected subgraph. We first prove that Bob can never win, and define a large class of graphs (called *reflection graphs*) in which the game is a draw. We then show that determining the outcome of the game is PSPACE-complete, even in bipartite graphs of small diameter, and that recognising reflection graphs is GI-hard. We also prove that the game is a draw in paths if and only if the path is of even order or has at least 11 vertices, and that Alice wins in cycles if and only if the cycle is of odd length. Lastly, we give an algorithm to determine the outcome of the game in cographs in linear time. This is joint work with Julien Bensmail, Foivos Fioravantes, and Nicolas Nisse.

Daniela Ferrero, Texas State University — Friday 11:00 - 11:30
Product Throttling Power Domination

Power domination is a particular form of graph searching introduced as a model for the monitoring process of electrical power networks. As electrical power systems become increasingly more complex, so does their monitoring process. As a result, new problems on power domination in graphs have recently been proposed. One of these problems is the study of product throttling for power domination, defined as a measure of the trade-off between time and cost involved in a power domination process. In this talk, we present recent results on the power throttling number of a graph, including the determination of this parameter for specific classes of graphs, as well as bounds and extremal values for the product throttling number of general graph.

Brendan Sullivan, Emmanuel College — Friday 11:30 - 12:00
Two Graph Products (Cartesian and Strong) and the Cops and Robber Game

This talk will share a few related ideas about graph products and the cops and robber game. First, I will share some results and conjectures about how the cop numbers of G and H relate to the cop numbers of $G \square H$ (Cartesian product) and $G \boxtimes H$ (Strong product), for both the "ordinary" (all cops can move per turn) and "lazy" (only one cop can move per turn) variants. Second, I will describe how these two graph products play starring roles in some SageMath code that student researchers and I have used to calculate cop numbers and test conjectures. (The code is based on algorithms described in two sources: the book by Bonato & Nowakowski and a paper by Clarke & MacGillivray).

Shahin Kamali, University of Manitoba — Friday 1:30 - 2:00

Algorithms for Burning Graph families

Graph burning is a simple model for the spread of social influence in networks. The objective is to measure how quickly a fire, e.g., a piece of fake news, can be spread in a network. The burning process takes place in discrete rounds. In each round, a new fire breaks out at a selected vertex, while the old fires extend to their neighbours and burn them. A burning schedule selects where the new fire breaks out in each round, and the burning problem asks for a schedule that burns all vertices in a minimum number of rounds, termed the burning number of the graph. In this talk, we review some recent algorithmic results for burning families of graphs that include graphs of bounded path-wdith, tree-width, path-length, and certain families of dense graphs.

Seyyed Aliasghar Hosseini, Simon Fraser University — Friday 2:00 - 2:30

W-Active Cops and Robber

The game of cops and robber is a rather old and well-known game played on graphs. There are several variations of the game, but the basic idea is that cops and robber move in the graph, the cops' goal is to capture the robber and the robber's goal is to prevent this from happening. Definitions of moving and capturing changes in different versions. In the basic version, there exists a loop on each vertex. This allows the cops or the robber to stay on their current vertex (by using the loop). Active cops and robber is a variation of the game where the graph is loopless. This prevents the cops and the robber from staying on a vertex. The key observation is that when the graph does not have loops on all of its vertices, then the set of one-cop-win graphs does no match with dismantlable graphs. In this talk, we introduce a new version, w-active, that resolves this issue, and we present some results and examples.

Participants

Deepak Bal (Montclair State University)

Anthony Bonato (Ryerson University)

Boris Brimkov (Slippery Rock University)

Christy Bui (Rutgers University)

Andrea Burgess (University of New Brunswick Saint John)

Josh Carlson (Williams University)

Nancy Clarke (Acadia University)

Danielle Cox (Mount Saint Vincent University)

Ryan Cushman (Western Michigan University)

Danny Dyer (Memorial University of Newfoundland)

Sean English (University of Illinois at Urbana-Champaign)

Jessica Enright (University of Glasgow)

Joshua Erde (Graz University of Technology)

Daniela Ferrero (Texas State University)

Stephen Finbow (St. Francis Xavier University)

Alan Frieze (Carnegie Mellon University)

Jesse Geneson (San Jose State University)

Karen Gunderson (University of Manitoba)

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