

RYERSON UNIVERSITY
DEPARTMENT OF MATHEMATICS
GRAPHS AT RYERSON (G@R) SEMINAR

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Date: Friday, February 9, 2018
 Time: 3pm
 Location: ENG 210

A Partition Problem for the Direct Sum of Cyclic Groups

Abstract:

Let $1 = n_0 < n_1 < \dots < n_t$ be odd integers and let $G = \bigoplus_{i=1}^t \mathbb{Z}_{n_i}$ be the direct sum of t cyclic groups of order n_1, n_2, \dots, n_t . Given $t + 1$ non-negative integers, $\alpha_0, \alpha_1, \dots, \alpha_t$, such that $\sum_{i=0}^t \alpha_i = \prod_{i=1}^t n_i$, does there exist a permutation φ of G such that

$$\left| \{x \in G \mid \varphi(x) - x \text{ has order } n_i\} \right| = \alpha_i \tag{1}$$

for every $i \in \{0, 1, \dots, t\}$? For example, let $(n_0, n_1, n_2) = (1, 3, 5)$ and $(\alpha_0, \alpha_1, \alpha_2) = (2, 8, 5)$. A permutation φ of $G = \mathbb{Z}_3 \times \mathbb{Z}_5$ satisfying condition (1) is given below, where the element $(i, j) \in G$ is denoted by ij :

x	00	10	20	01	11	21	02	12	22	03	13	23	04	14	24
$\varphi(x)$	00	11	21	01	10	22	12	02	20	13	23	03	14	24	04
$\varphi(x) - x$	00	01	01	00	04	01	10	20	03	10	10	10	10	10	10
order	1	5	5	1	5	5	3	3	5	3	3	3	3	3	3

In this talk, we show that a permutation that satisfies property (1) can be obtained by constructing a suitable partition of G . We also show that this permutation can be used to decompose the complete equipartite graph $K_t[w]$ – with t parts of size w – into copies of t distinct “uniform” 2-regular graphs of order tw .

ALL FACULTY, STAFF, STUDENTS AND GUESTS ARE WELCOME TO ATTEND