

RYERSON UNIVERSITY
DEPARTMENT OF MATHEMATICS
GRAPHS AT RYERSON (G@R) SEMINAR

Dr. Dejan Delic

Department of Mathematics, Ryerson University

Date: Monday, March 27, 2017

Time: 10am

Location: ENG 210

Space Dichotomy for List Homomorphism Problems

Abstract:

For a fixed finite relational structure \mathbb{A} its *list homomorphism problem* $\text{LHOM}(\mathbb{A})$ asks if, given a similar structure $\mathbb{X} = \{x_1, \dots, x_n\}$ and n subsets $L_1, \dots, L_n \subseteq A$ whether there exists a homomorphism $f : X \rightarrow A$ such that $f(x_i) \in L_i$. As a constraint satisfaction problem, it corresponds to the case where every polymorphism $h : A^k \rightarrow A$ has the *conservative property*: $h(a_1, \dots, a_k) \in \{a_1, \dots, a_k\}$. The intricate proof by Bulatov (2006) shows that the Dichotomy Conjecture holds for list homomorphism problems over finite templates.

The Space Dichotomy, formulated by Larose and Tesson (2009) states that every tractable CSP (not necessarily a conservative one) is either in logspace or NL-hard. In fact, they conjectured a finer classification result; namely, every CSP of bounded relational width (i.e. expressible in unrestricted Datalog) is either expressible in Symmetric Datalog, or Linear Datalog or is P-complete. This provides full classification of all CSPs of the bounded relational width from the standpoint of descriptive complexity. In its full generality, this problem is still open. In 2013, Egri, Hell, Larose, and Rafiey proved that the Space Dichotomy holds for directed graph templates, while giving a nice graph theoretic characterization of the problems solvable in logspace which, itself, can be checked in polynomial time. This result was refined by the same authors and Dalmau in 2015 by generalizing it to the templates with a finite number of binary relations.

In this talk, we will show how to generalize the List Homomorphism Space Dichotomy to *all* finite templates and, in the process, obtain a stronger result; namely, the Symmetric and Linear Datalog Conjectures hold. We use the fine analysis of polymorphisms on 2-element subsets of the structure which gives rise to a 3-coloured digraph whose certain monochromatic induced subgraphs provide information about “nice global properties which guarantee the placement in the presumptive complexity class.

ALL FACULTY, STAFF, STUDENTS AND GUESTS ARE WELCOME TO ATTEND