On the density of nearly regular graphs with a good edge-labelling

Abstract: A good edge-labelling of a simple graph is a labelling of its edges with real numbers such that, for any ordered pair of vertices $(u, v)$, there is at most one nondecreasing path from $u$ to $v$. Say a graph is ‘good’ if it admits a good edge-labelling, and is ‘bad’ otherwise. Our main result is that any good $n$-vertex graph whose maximum degree is within a constant factor of its average degree (in particular, any good regular graph) has at most $n^{1+o(1)}$ edges. As a corollary, we show that there are bad graphs with arbitrarily large girth, answering a question of Bode, Farzad and Theis. We also prove that for any $\Delta$, there is a $g$ such that any graph with maximum degree at most $\Delta$ and girth at least $g$ is good. The paper is available via http://arxiv.org/abs/1110.2391.