2nd Graph Searching in Canada (GRASCan) Workshop

April 26–28, 2013
Ryerson University
Toronto, Canada

RYERSON UNIVERSITY
Everyone Makes a Mark

Friday, April 26

9:15 - 9:30	Opening remarks
9:30 - 10:30	**Richard Nowakowski**, Dalhousie University *Location, Location, Location*
10:30 - 11:00	Coffee break
11:00 - 12:00	Problem Session I
12:00	Lunch

Saturday, April 27

9:30 - 10:00	**Bill Kinnersley**, Ryerson University *The capture time of the hypercube*
10:00 - 10:30	**Masood Masjoody**, Simon Fraser University *The Game of Cops and Robber in two Classes of UDG's*
10:30 - 11:00	Coffee break
11:00 - 11:30	**David A. Pike**, Memorial University of Newfoundland *Brushing without capacity restrictions*
11:30 - 12:00	**Boting Yang**, University of Regina *The Optimal Capture Time of a Cops-and-Robber Game*
12:00	Lunch

Sunday, April 28

9:30 - 10:00	**Przemysław Gordinowicz**, Technical University of Łódź *Fire! Planar graph is on fire!*
10:00 - 10:30	**Natasha Komarov**, Dartmouth College *Capturing the Drunk Robber on a Graph*
10:30 - 11:00	Coffee break
11:00 - 11:30	**Kerry Ojakian**, Bronx Community College (CUNY) *Cops and Robber on the Hypercube*
11:30 - 12:00	**Paweł Prałat**, Ryerson University *Cops and Robbers playing on edges*
12:00	Lunch

Participants

Robert Bailey (Ryerson)
Kathleen Barnetson (Memorial)
Andrew Beveridge (Macalester)
Anthony Bonato (Ryerson)
Andrei Bulatov (SFU)
Andrea Burgess (Ryerson)
Nancy Clarke (Acadia)
Peter Danziger (Ryerson)
Stephen Finbow (St. Francis Xavier)
Przemyslaw Gordinowicz (Lodz)
Gena Hahn (Montreal)
Bill Kinnersley (Ryerson)
Natasha Komarov (Dartmouth)
Masood Masjoody (Simon Fraser University)
Margaret-Ellen Messinger (Mount Allison)
Pardis Noorzad (Ryerson)
Richard Nowakowski (Dalhousie)
Kerry Ojakian (Bronx Community College (CUNY))
David Pike (Memorial)
Vivija Ping You (Ryerson)
Pawel Pralat (Ryerson)
Ben Seamone (Montreal)
Ladislav Stacho (Simon Fraser University)
Carrie Wang (SFU)
Boting Yang (Regina)

Abstracts of Talks

Richard Nowakowski, Dalhousie Univeristy — Friday, 9:30 - 10:30
Location, Location, Location

With perfect information the Robber's position is known at all times. In the imperfect information case, there is a theme of using 'technology' (e.g., alarms, cameras) to help first infer where the location of the robber then capture then Robber, even if his location isn't known exactly during this second phase. I'll survey the known results and some of the new problems.

Bill Kinnersley, Ryerson University — Saturday, 9:30 - 10:00
The capture time of the hypercube

In the classic "Cops and Robber" game, a team of cops attempts to capture a robber on a graph; the *cop number* of the graph is the minimum number of cops needed to guarantee a win. The cop number has been the subject of numerous papers and has spawned many variants. In this talk, we consider a natural question that has received surprisingly little attention: how long does it take for the cops to catch the robber? When both teams play optimally, the length of the game is called the *capture time*. After briefly surveying previous work, we show that the capture time of the n-dimensional hypercube is $\Theta(n \log n)$.

This is joint work with Anthony Bonato, Przemysław Gordinowicz, and Paweł Prałat.

Masood Masjoody,Simon Fraser University — Saturday, 10:00 - 10:30
The Game of Cops and Robber in two Classes of UDG's

It is known that finite unit distance graphs (UDG's) have cop number at most 9. We show that in the class of UDG's which do not contain an induced subgraph isomorphic to a claw or a 3-cycle augmented by two independent edges each incident to a vertex of the cycle, the upper bound of 9 can be reduced to 6. Our idea of proof also works for proving the same result for the class of UDG's excluding an induced subgraph isomorphic to a claw augmented by an edge between two of its vertices. Our proofs rely on the fact that in the UDG's under consideration every geodesic path can be guarded by two cops.

This is a joint work with Ladislav Stacho.

David A. Pike, Memorial University of Newfoundland — Saturday 11:00 - 11:30
Brushing without capacity restrictions

We consider a variant of the problem of cleaning a graph with brushes, whereby one vertex is cleaned at a time and there is no restriction on the number of brushes that are permitted to traverse an uncleaned edge. Given a graph G, the main question of interest is to determine its brushing number $B(G)$, i.e., the minimum number of brushes that enable the graph to be cleaned. We obtain results for trees and Cartesian products, as well as general upper and lower bounds on the brushing number.

This is joint work with Darryn Bryant, Nevena Francetić, Przemysław Gordinowicz and Paweł Prałat.

Boting Yang, University of Regina — Saturday 11:30 - 12:00
The Optimal Capture Time of a Cops-and-Robber Game

Given a graph with a set of cops and a single robber located on vertices, in each round of the game, the robber moves to an adjacent vertex and then one of the cops moves to his adjacent vertex. We want to find a strategy for cops that minimizes the time taken by the cops to capture the robber. In particular, we explore algorithms regarding this game on trees involving two cops and a single robber. We present four algorithms that compute optimal-search, monotonic-search, greedy-search and balanced-search strategies respectively. We show performance ratios when some of them are considered as approximation algorithms. We discuss the different applications depending on cops' visibility.

Joint work with William Hamilton.

Przemysław Gordinowicz, Technical University of Łódź — Sunday 9:30 - 10:00
Fire! Planar graph is on fire!

Let G be any connected graph on n vertices, $n \geq 2$, and let k be any positive integer. Suppose that a fire breaks out on some vertex of G. Then in each turn, k firefighters can protect some vertices of G; each firefighter can protect one vertex not yet on fire. Next, a fire spreads to all not protected neighbours.

The *k-surviving rate* of G, denoted by $\rho_k(G)$, is the expected fraction of vertices that can be saved from the fire by k firefighters, provided that the starting vertex is chosen uniformly at random. During this talk, it will be shown that for any planar graph G we have $\rho_3(G) \geq 2/21$. Moreover, 3 firefighters are needed for the first step only; after that it is enough to have 2 firefighters per each round. This result significantly improves known solutions of a problem of Cai and Wang (there was not known any positive bound for surviving rate of general planar graph with only 3 firefighters).

Natasha Komarov, Dartmouth College — Sunday 10:00 - 10:30

Capturing the Drunk Robber on a Graph

We consider a variation on the cops and robbers pursuit-evasion game in which the robber proceeds according to a random walk, and address the question of (expected) capture time. It turns out that the drunk can be captured in $n+o(n)$ steps by a somewhat intelligent cop. We will motivate this result by showing examples of graphs that confound cops that follow a couple of obvious strategies. Time permitting, we will briefly discuss the intuition behind the proof of this result.

Kerry Ojakian, Bronx Community College (CUNY) — Sunday 11:00 - 11:30

Cops and Robber on the Hypercube

We determine the cop number of the hypercube for different versions of the game Cops and Robber. The n-dimensional hypercube is the graph whose vertices are the length n binary vectors with an edge between vectors that differ at exactly one coordinate. We consider cops of varying capacities: We refer to a cop as active if she must move, passive if she must remain still, and flexible if she may move or stay still. By varying the number of flexible, active, and passive cops we consider a whole spectrum of games. We fully classify the trade-off between active and flexible cops. Introducing passive cops dramatically increases the difficulty and interest of the problem. I will discuss some of these results and present some open questions.

This is joint work with David Offner.

Paweł Prałat, Ryerson Universtiy — Sunday 11:30 - 12:00

Cops and Robbers playing on edges

In the game of cops and robber, the cops try to capture a robber moving on the vertices of the graph. The minimum number of cops required to win on a given graph G is called the cop number of G. This time, we consider the variant of the game in which both players play on edges instead of vertices, yielding the edge cop number. We relate the new graph parameter to the classic one, investigate Meyniel extremal families, and characterize edge cop-win graphs. We also play the game on random graphs and planar graphs.

This is joint work with Andrzej Dudek and Przemysław Gordinowicz.